Year 12 Homework Booklet – Summer 2021

Subject: Pure Mathematics
Year 12 Single Maths: Volumes of Revolution

Name: ...........................................................................................................

Class: ...........................................................................................................

Teacher: ........................................................................................................

Homework Checklist

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The work is compulsory if you wish to get a place on the A-Level Maths course at Oasis Academy South Bank Sixth Form. There are six packs of work, one for each week. You should spend at least one hour on each pack.

Write your working out and solutions in this booklet. To self-assess your work, follow the links on the Oasis Academy South Bank website (SIXTH FORM -> ADMISSIONS -> SUMMER PROJECTS).

Your A-Level maths teacher will check your work in September.

If you would like more maths work, or have any questions about the A-Level Mathematics course, please email dominic.williams@oasissouthbank.org or danielle.foster@oasissouthbank.org.

Further mathematicians should also complete the work in this pack. If they wish to do further work, please read the instructions on the website.
Factorise the following fully:

1. \( x^2 + 5x - 6 \)  
5. \( k^2 - 2k - 24 \)

2. \( x^2 + 13x - 30 \)  
6. \( p^2 - 10p + 21 \)

3. \( y^2 - 13y + 30 \)  
7. \( x^2 - 16x \)

4. \( t^2 + 2t - 15 \)  
8. \( 3x(2x - 1) + 4(1 - 2x) \)
2. Difference of two squares

Try factorising these expressions using the difference of two squares

1. \(x^2 - 6^2\)
2. \(y^2 - 144\)
3. \(x^2 - y^2\)
4. \(4t^2 - 81\)
5. \(x^2 - 5\)

3. Trickier quadratics

- Try factorising these expressions
- You might want to try the grid method.

1. \(3x^2 - 10x - 8\)
2. \(2x^2 - 7x + 6\)
3. \(4y^2 + 20y + 9\)
4. \(6x^2 - 13x - 8\)
5. \(20x^2 + x - 12\)
These expressions are slightly different to the previous ones, but can still be factorised.

1. \(2t^2 - 32\)

2. \(x^3 - 7x^2 + 12x\)

3. \(x^4 - x^2 - 2\)

4. \(y^4 - 625\)

4. **Difference of two squares problems**

**What is the value of each of the following? calculators not allowed**

\[
9^2 - 1^2 \\
99^2 - 1^2 \\
999^2 - 1^2
\]

Without using a calculator, find the value of

\[
\frac{122 \times (122^2 + 4 \times 123)}{124} - \frac{124 \times (124^2 - 4 \times 123)}{122}
\]
Pack 2 – Completing the square

1. Completing the square

Write these expressions in the form \((x + a)^2 + b\)

1. \(x^2 + 4x\)
2. \(x^2 + 4x + 5\)
3. \(y^2 - 8y\)
4. \(y^2 - 8y + 7\)
5. \(x^2 - 12x + 41\)
6. \(k^2 + 10k - 2\)
7. \(y^2 + 3y + 1\)
8. \(p^2 - 2p + 1\)
2. Different forms

It is important to be able to convert expressions between the different forms:

<table>
<thead>
<tr>
<th>expanded form</th>
<th>factorised form</th>
<th>completed square form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a^2 - 2a - 8$</td>
<td>$a^2 + 2a - 15$</td>
<td>$(a + 2)(a + 4)$</td>
</tr>
<tr>
<td>$(a + 1)^2 - 16$</td>
<td>$(a - 3)(a - 5)$</td>
<td></td>
</tr>
<tr>
<td>$(a + 5)(a - 3)$</td>
<td>$(a - 1)^2 - 9$</td>
<td>$(a + 3)^2 - 1$</td>
</tr>
</tbody>
</table>

In this problem there are 4 sets of three equivalent expressions, however, some expressions are missing. Match the sets and find the 3 missing expressions.
3. Extra puzzles (non-calculator!!)

What is the value of

\[
\frac{(5^2-3^2)}{5+3} + \frac{(4^2-2^2)}{4+2} + \frac{(3^2-1^2)}{3+1}
\]

Given that

\[
55^2 - 45^2 = (55 + 45)(55 - 45) = 1000
\]

and

\[
60^2 - 40^2 = (60 + 40)(60 - 40) = 2000
\]

Find numbers \(a\) and \(b\) such that \(a^2 - b^2 = 3000\)

Find numbers \(c\) and \(d\) such that \(c^2 - d^2 = 4000\)

Find numbers \(e\) and \(f\) such that \(e^2 - f^2 = 100000\)
Pack 3 – Rearranging fractions

1. **Rearranging fractions (write your solutions BELOW)**

1. Rewrite the formula below to make time the subject.
   \[ \text{Speed} = \frac{\text{Distance}}{\text{time}} \]

2. Rearrange to make \( a \) the subject of \( \frac{x}{y} = \frac{a}{b} \)

3. Make \( x \) the subject of \( \tan \theta = \frac{y}{x} \)

4. These triangles are similar. Show that \( x = \frac{cb}{a} \)

5. Make \( a \) the subject of \( x = \frac{h+k}{a} \)

6. Make \( x \) the subject of \( x + a = \frac{x+b}{c} \)

7. Make \( a \) the subject of \( \frac{z-a}{1+a} = \frac{x}{y} \)

8. Make \( x \) the subject of \( y(\sqrt{a} + \sqrt{b}) = x \) and write it in the form \( x(\sqrt{a} + \sqrt{b}) \)
2. **Rearranging fractions 2**

1. Make \( x \) the subject of 
   \[
   bc = \frac{x}{a}
   \]

2. Make \( e \) the subject of 
   \[
   x = \frac{y}{e^2}
   \]

3. Write \( a \) in terms of \( x, y, z \) and \( b \) 
   \[
   \frac{b-xo}{z} = y
   \]

4. Make \( v \) the subject of 
   \[
   C = \frac{v^2-ea}{x}
   \]

5. Rearrange to make \( x \) the subject of 
   \[
   \frac{2}{x} + 5 = 6y
   \]

6. Make \( x \) the subject of 
   \[
   4F = F + \frac{a}{y+x}
   \]

7. Make \( y \) the subject of 
   \[
   \sqrt{\frac{m(y+a)}{y}} = g
   \]

8. A cylinder has a radius of 3cm and height, \( h \). The total surface area = 30\( \pi \) cm\(^2\).

   Find an expression for the surface area and write \( h \) in terms of \( x \) and \( \pi \).
3. **Wrong steps**

Each expression has been rewritten in different ways.

- Which are not correct rearrangements?
- Give your reasons

\[
c = \frac{3e^2}{d}
\]

<table>
<thead>
<tr>
<th>A. ( d = 3e^2 - c )</th>
<th>B. ( cd = 3e^2 )</th>
<th>C. ( \frac{d}{e^2} = \frac{c}{3} )</th>
<th>D. ( \frac{1}{3}c = \frac{e^2}{d} )</th>
<th>E. ( d = \frac{3e^2}{c} )</th>
</tr>
</thead>
</table>

\[
\frac{\sin x}{4} = \frac{\sin y}{a}
\]

<table>
<thead>
<tr>
<th>A. ( \frac{a}{4} = \frac{\sin y}{\sin x} )</th>
<th>B. ( \sin y = \frac{4}{a \sin x} )</th>
<th>C. ( \sin x = \frac{4\sin y}{a} )</th>
<th>D. ( a \sin x = 4\sin y )</th>
<th>E. ( a = \frac{\sin x}{4\sin y} )</th>
</tr>
</thead>
</table>

\[
\frac{T - a}{T + a} = \frac{x}{y}
\]

<table>
<thead>
<tr>
<th>A. ( x(T + a) = y(T - a) )</th>
<th>B. ( xy - ay = yT - ya )</th>
<th>C. ( a = \frac{y(T - a)}{x+y} )</th>
<th>D. ( xa + ya = yT - xT )</th>
<th>E. ( a = \frac{x+y}{yT - ya} )</th>
</tr>
</thead>
</table>

Each expression has been rewritten in different ways.

- Which are not correct rearrangements?
- Give your reasons

\[
a - \frac{b^2}{d} = ce
\]

<table>
<thead>
<tr>
<th>A. ( b^2 = d(a + ce) )</th>
<th>B. ( a = ce + \frac{b^2}{d} )</th>
<th>C. ( \frac{b^2}{d} = a - ce )</th>
<th>D. ( \frac{b}{\sqrt{d}} = \sqrt{a} - \sqrt{ce} )</th>
<th>E. ( b = \pm \sqrt{d(a - ce)} )</th>
</tr>
</thead>
</table>

\[
y + b = \frac{ay + e}{b}
\]

<table>
<thead>
<tr>
<th>A. ( by + b^2 = ay + e )</th>
<th>B. ( by - ay = e + b^2 )</th>
<th>C. ( y = \frac{e - b^2}{b - a} )</th>
<th>D. ( e = b(y + b) - ay )</th>
<th>E. ( y(b - a) = \frac{e - b^2}{y} )</th>
</tr>
</thead>
</table>
4. Prove it! (write your solutions below!!)

Using your rearranging skills can you prove each of the following

If \[ a = \frac{b}{b+c} \]

Show that \[ \frac{a}{1-a} = \frac{b}{c} \]

\[ \frac{n(n-1)}{2} + \frac{n(n+1)}{2} \] is a square number

\[ \frac{2x + 3}{4} - \frac{3x - 2}{3} + \frac{1}{6} = \frac{19 - 6x}{12} \]
Pack 4 – Sketching linear graphs

1. **Straight line graphs** 1

1. What are the gradient and intercept of the line \( y = 3x - 5 \)

2. Find the gradient of the line connecting (3,10) and (1,6)

3. Find the midpoint between the points (3,-8) and (-1,4)

4. Find the distance between points (1,10) and (4,18)

5. What is the equation of the line with gradient 3 that passes through (5,8)?

6. Does the line \( y = 2x - 3 \) pass through (1,-1)? Explain how you know.

7. Find the equation of a line that is parallel to \( y = 5x - 2 \) that passes through (2,19)

8. What is the equation of this graph?
2. **Straight line graphs 2**

1. What are the gradient and y intercept of the line $y = 2x - 7$

2. Find the gradient of the line connecting (1,4) and (-1,0)

3. Find the midpoint between the points (-2,10) and (6,4)

4. Find the distance between the points (4,11) and (-1,15)

5. What is the equation of the line with gradient 2 that passes through (1,4)?

6. Does the line $y = -2x + 5$ pass through (3,1)? Explain how you know.

7. Find the equation of a line that is parallel to $y = -\frac{3}{2}x - 1$ that passes through (6,4)

8. What's the equation of this graph?
3. Do they cross?

Line A passes through the points \((-3,1)\) and \((3,5)\)

Line B passes through the points \((0,-4)\) and \((6,4)\)

- By sketching can you tell if the lines will meet?
- If they do meet what the points of intersection?
4. Two geometry problems

DEF is an isosceles right angled triangle

- The line passing through D and F has the equation $x + 3y = 15$
- D is the co-ordinate (6,3)
- E is the co-ordinate (5,0)
- The angle EDF is the right angle

Can you find:
- The equation of line DE?
- The possible coordinates of F?
- The equation of line EF?

ABCDD is a parallelogram

- The line passing through C and D has the equation $y = 7$
- The line CD is 5 units long
- D has coordinate (2,7)
- C has both positive x and y co-ordinates
- The line through AC has equation $3x + 2y = 35$
- A has coordinate (9,4)

Can you find:
- The coordinate of C?
- The equation of line AB?
- The equation of line BD?
- The area of the parallelogram?

Hint: Sketch the graphs!!
5. **Geometry from equations**

The following equations enclose a square:

\[
\begin{align*}
y - 2 &= x \\
y + x &= 6 \\
y &= x - 1 \\
y + x - 3 &= 0
\end{align*}
\]

- Which are the two pairs of parallel sides?
- What are the coordinates of all 4 vertices?
- How can you convince yourself this is a square?
6. Sketching linear inequalities

- Sketch and shade the following inequalities.

1. \( y \leq 6 \)
2. \( x < 6 \)
3. \( x + 2y \geq 8 \)
4. \( 3x + 2y \geq 12 \)

- Shade out the side of the line that doesn’t satisfy the inequality.
- Label the correct region \( R \)
Pack 5 – Sketching Quadratics

1. Sketching quadratics

Find the coordinates of A, B and C etc. on each graph

\begin{align*}
  y &= (x + 3)(x - 2) \\
  y &= (x - 3)(x + 2) \\
  y &= (x - 5)(x + 1) \\
  y &= (x + 4)(x - 5) \\
  y &= (x - 1)^2 + 4 \\
  v &= (x + 3)^2 + 7 \\
  v &= x^2 + 3x - 4 \\
  v &= 2x^2 + 4x - 6
\end{align*}
2. **Key features**

These diagrams show the key features of a quadratic graph

\[ y = x^2 + 4x + 3 \]

\[ y = -x^2 - 4x + 1 \]

Put the words below into the boxes above so that the quadratic graphs are labelled correctly. Some words may be used more than once.

- \(x\) intercepts
- minimum
- roots
- turning point
- maximum
- axis of symmetry
- \(y\) intercept

3. **Match up!**

- Which of the following graphs is \( y = x^2 - 5x + 4 \)
4. Move it!

- Can you describe how to move Graph A onto Graph B?

  ![Graphs A and B with the equation \( y = x^2 \) and \( y = x^2 - 1 \), and the vector \((0, -1)\).]

- Which transformations would take Graph A onto each of the graphs below?

  ![Graphs C, D, and E with the equations \( y = x^2 + 3 \), \( y = (x - 4)^2 \), and \( y = (x - 2)^2 \).]

- Can you describe how to move Graph A onto Graph B?

  ![Graphs A and B with the equation \( y = x^2 \) and \( y = (x - 4)^2 + 3 \).]

- Can you see how that links to the equation of the graph?
5. Quadratic graphs 2

1. What are the $x$ intercepts of \[ y = (2x + 3)(x + 4) \]

2. What are the $x$ and $y$ intercepts of this graph.

3. Write the equation of the graph in the form $ax^2 + bx + c$

4. What are the $x$ intercepts of the graph of \[ y = 6x^2 + x - 2 \]

5. What does the $c$ part of the equation in \[ y = ax^2 + bx + c \] represent on a graph?

6. Sketch the graph of $y = 3x^2 - 2x - 8$. Label $x$ and $y$ intercepts

7. What are the co-ordinates of the points marked on the diagram of the equation \[ y = x^2 + 6x + 16 \]

8. Which of these statements about the graph $y = x^2 - 4x + 8$ are true

   - Has a minimum point at (2, 4)
   - Will not cross the $x$ axis twice
   - Can be factorised
6. How high? (use the whole page!)

The height of a ball thrown up from the ground into the air at time, $t$, is given by:

$$ h = 20t - 10t^2 $$

- Find when the ball hits the ground
- How long is the ball more than 5m above the ground?
- Find the maximum height reached by the ball
7. Quadratic inequalities – make sure you draw a sketch and think about what the inequality is saying!

Use a sketch to help you solve the following inequalities

1. \((x - 2)(x + 3) < 0\)
2. \((4 + x)(2 - x) < 0\)
3. \(x^2 + 7x + 12 \geq 0\)
4. \(36 \geq (x + 2)^2\)
8. Fill in the table:

<table>
<thead>
<tr>
<th>Sketch</th>
<th>Equation</th>
<th>$x$ intercept</th>
<th>$y$ intercept</th>
<th>Minimum point</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Sketch" /></td>
<td>$(5, 0), (-2, 0)$</td>
<td>$(0, -10)$</td>
<td></td>
<td>$(-5, 6)$</td>
</tr>
<tr>
<td></td>
<td>$y = x^2 + 6x + 8$</td>
<td></td>
<td>$(0, 8)$</td>
<td></td>
</tr>
</tbody>
</table>
Pack 6 – Sketching other graphs – WRITE YOUR SOLUTIONS BELOW!!

1. Which is which?

Match the graphs to the equations There are more equations than you need!

- $y = 3^x$
- $y = \frac{3}{x}$
- $y = \sin \theta$
- $y = 2x^2$
- $y = \tan \theta$
- $y^2 + x^2 = 4$
- $y = x^2$
- $y = x^3$
- $y = \frac{1}{x}$
- $y = x^2 + 3$
2. **Sketching graphs 1**

1. What is the mathematical name for the graph of \( y = \frac{1}{x} \)?

2. What are the maximum and minimum values for the graph \( y = \cos \theta \)?

3. Sketch the graph of \( y = 2^x \).
   Label the \( y \) and \( x \) intercepts.

4. Using a sketch of the graphs \( y = \frac{1}{x} \) and \( y = x \)
   show how many solutions there will be to the equation \( \frac{1}{x} = x \)

5. What is the name for this type of graph?

6. What is the \( y \) intercept of the graph \( y = (x + 2)(x - 3)(x + 5) \)?

7. What are the \( x \) intercepts of the graph \( y = (x + 2)(x - 3)(x + 5) \)?

8. Sketch the graph of
   \( y = (x - 3)(x + 2)(x + 5) \)
3. **Sketching graphs 2**

1. What is the mathematical name for graphs of the form of \( x^2 + y^2 = 9 \)?
2. Sketch the graph of \( y = \sin \theta \) between 0° and 360°, labelling x and y intercepts.
3. On your sketch for Q2 draw in the line \( y = 0.5 \).
   How many solutions are there to \( \sin \theta = 0.5 \)?
   Can you say what they are?
4. Sketch the graph \( y = x^3 \), labelling any intersections.
5. Sketch the graph of the equation in Q1, label any intersections with the x and y axis.
6. What is the y intercept of the graph \( y = (x + 1)(x + 1)(x - 1) \)?
7. What are the x intercepts of the graph \( y = (x + 1)(x + 1)(x - 1) \)?
8. Sketch the graphs of
   \[ x^2 + y^2 = 4 \]
   \[ y = x + 1 \]
   Use the sketch to determine how many solutions there are when those equations are solved simultaneously.
4. **Sketching more than graphs**

Find the shortest distance between the following curves:
\[ x^2 + y^2 = 9 \]
\[ y = x^2 + 7 \]

A car is initially travelling at 300m/min, it speeds up over a 20 second interval with a constant acceleration to achieve a speed of 400m/min. It travels at this speed for 3 minutes before slowing to a stop via constant deceleration over a period of 30 seconds.

a) What is the car’s average speed for the first 20 seconds of travel?
b) What is the car’s deceleration?

A square is placed inside a circle \( (C_1) \) so that the corners perfectly touch the circle’s circumference. Another circle \( (C_2) \) is now placed inside this square so that its circumference perfectly touches the square’s sides.

What is the ratio of the lengths of the radius of \( C_1 \) and the radius of \( C_2 \)?

*Hint: Assume \( C_2 \) has a radius of 1 unit*
5. **CHALLENGE: A triggy problem**

Solve \((\sin x + 1)(2\cos x - 1) = 0\) for \(0 < x < 360^\circ\)
6. **CHALLENGE:**

**Which one of the equations below describes the graph?**

- $y = (x + 1)(x - 1)(x - 2)$
- $y = -x(x - 1)(x + 1)$
- $y = x(x - 1)(x + 1)$