Advanced Mathematics Support Programme®

Did you know?

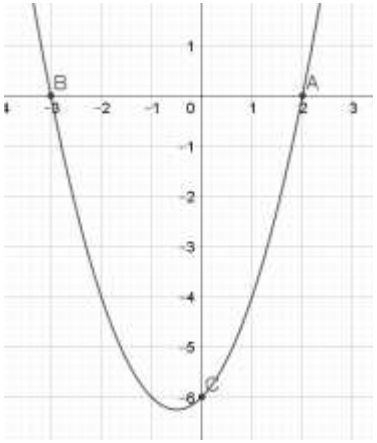
Quadratic curves are also known as parabolas.

Parabolas are used in many examples of architecture.

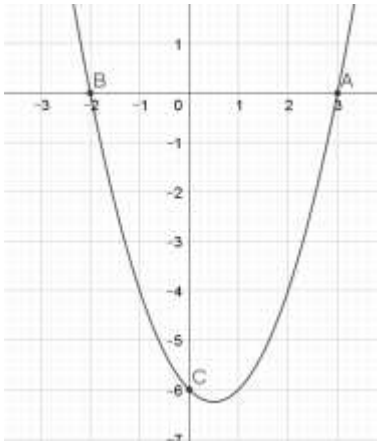
- Do you recognise these landmarks?
- Do you think they are parabolas?

Quadratic Graphs 1

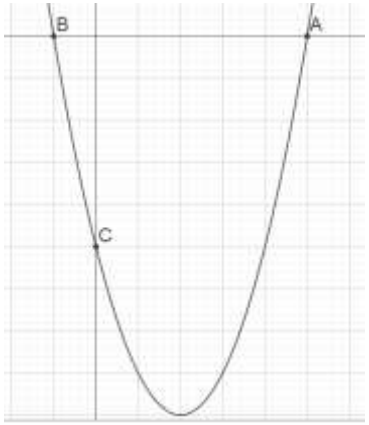
Find the coordinates of A, B and C etc.. on each graph



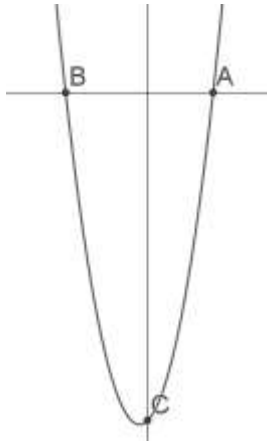
$y = (x + 3)(x - 2)$



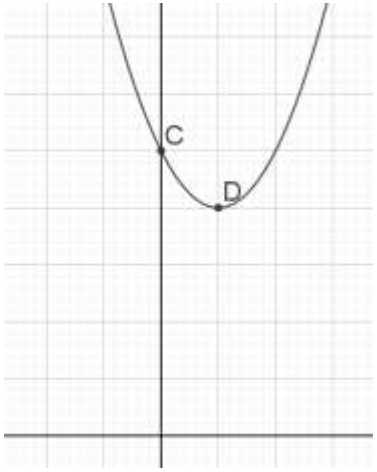
$y = (x - 3)(x + 2)$



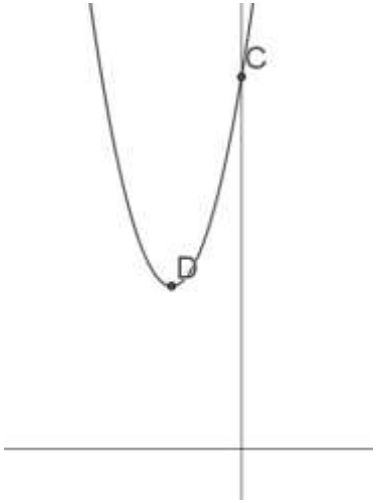
$y = (x - 5)(x + 1)$



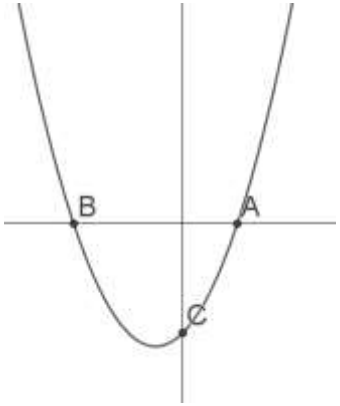
$y = (x + 4)(x - 5)$



$y = (x - 1)^2 + 4$



$y = (x + 3)^2 + 7$



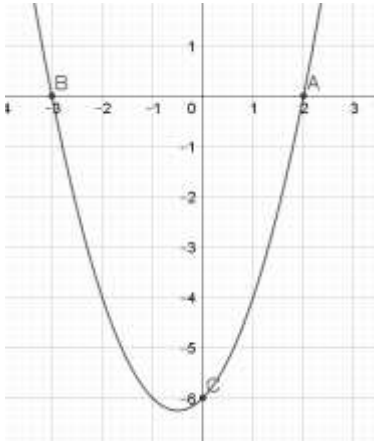
$y = x^2 + 3x - 4$

$y = 2x^2 + 4x - 6$

Quadratic Graphs 1

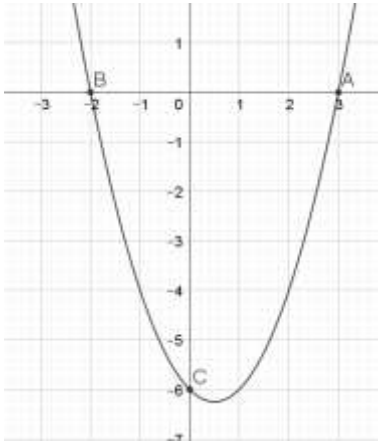
Solutions on the next slide….

Quadratic Graphs 1 Solutions



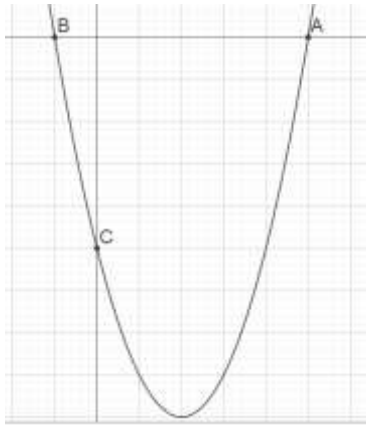
$y = (x + 3)(x - 2)$

A(2,0)
B(−3,0)
C(0, −6)



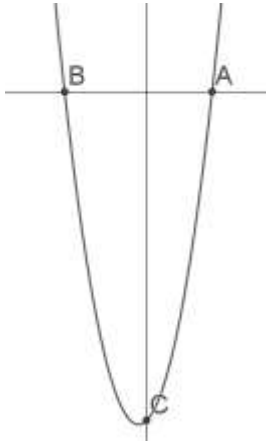
$y = (x - 3)(x + 2)$

A(3,0)
B(−2,0)
C(0, −6)



$y = (x - 5)(x + 1)$

A(5,0)
B(−1,0)
C(0, −5)



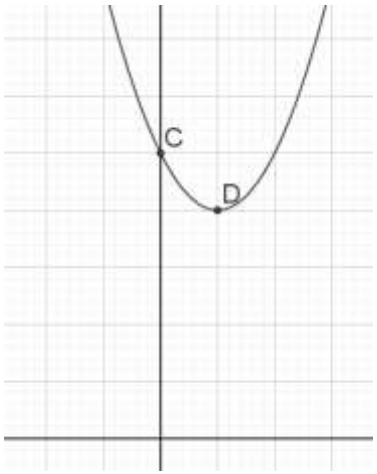
$y = (x + 4)(x - 5)$

A(−4,0)
B(5,0)
C(0, −20)

Unsure about any of these? Search Sketching Quadratic Graphs. Next try Quadratics Graphs 2….

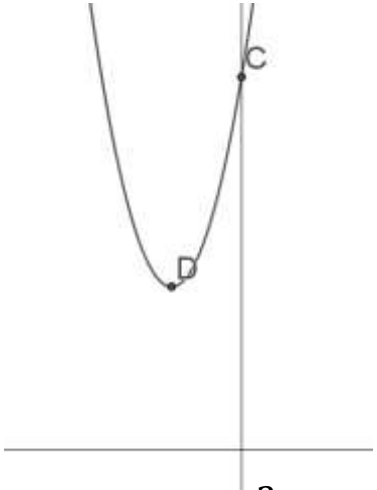
Quadratic Graphs 1 Solutions

For these two equations you need to factorise to find the roots of the equation A and B



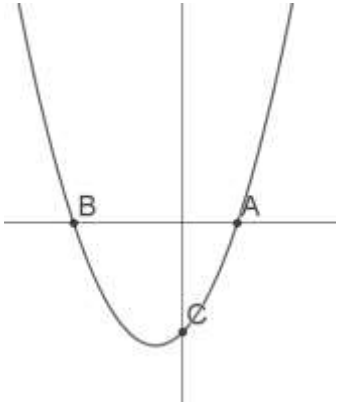
$y = (x-1)^2 + 4$

C(0,5)
D(1,4)



$y = (x+3)^2 + 7$

C(0,16)
D(−3,7)



$y = x^2 + 3x - 4$

A(1,0)
B(−4,0)
C(0, −4)

$y = 2x^2 + 4x - 6$

A(1,0)
B(−3,0)
C(0, −6)
D(−1, −8)

For this equation you need to complete the square to find the minimum point D

Unsure about any of these? Search Sketching Quadratic Graphs. Next try Quadratic Graphs 2….

What is a sketch?

In mathematics a sketch does not need to be a completely accurate drawing, but it should "illustrate all the significant features of the graph/shape"

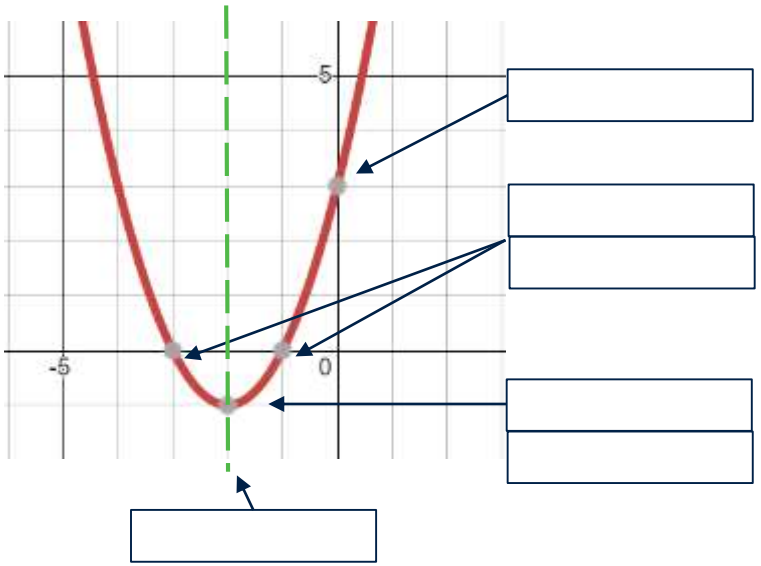
What do you think the significant or key features of a quadratic graph are?

What techniques have you used so far that can help you find those key features?

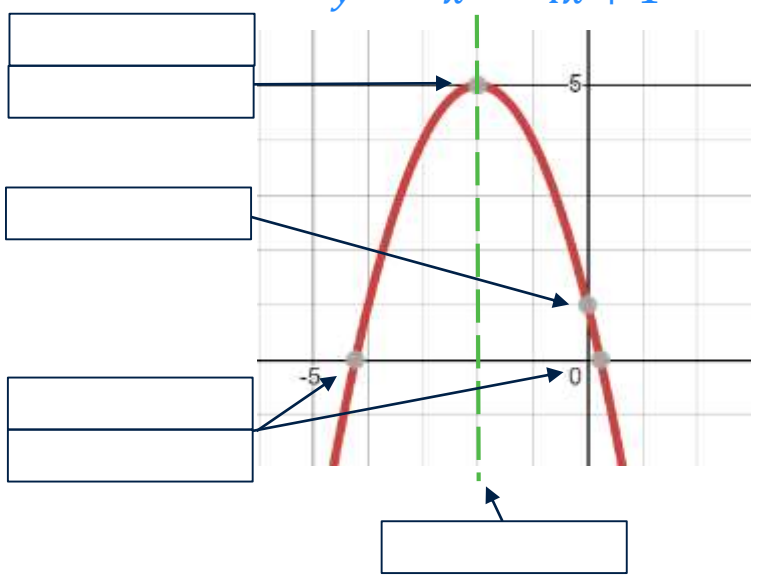
Label it!

These diagrams show the key features of a quadratic graph

$y = x^2 + 4x + 3$



$y = -x^2 - 4x + 1$



Put the words below into the boxes above so that the quadratic graphs are labelled correctly. Some words may be used more than once.

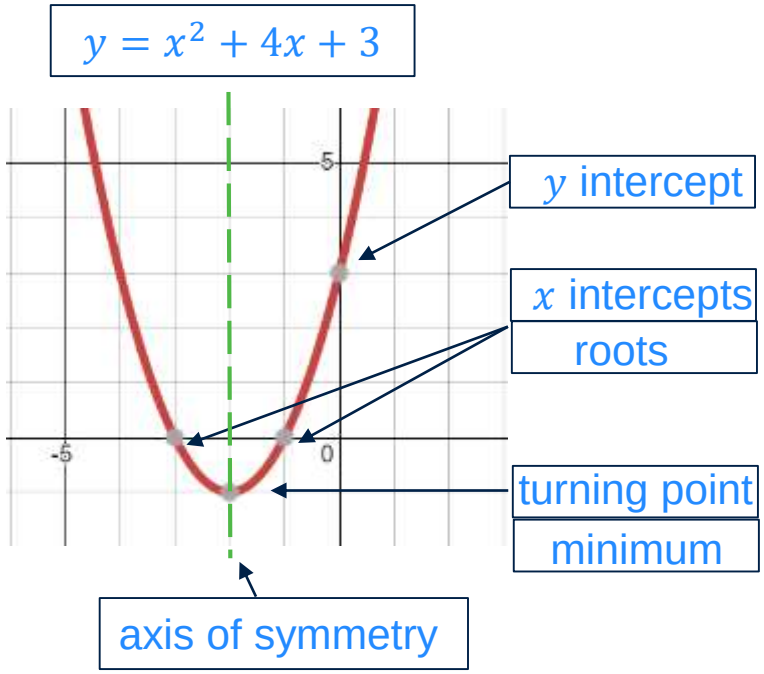
x intercepts	minimum	roots	turning point
maximum	axis of symmetry	y intercept	

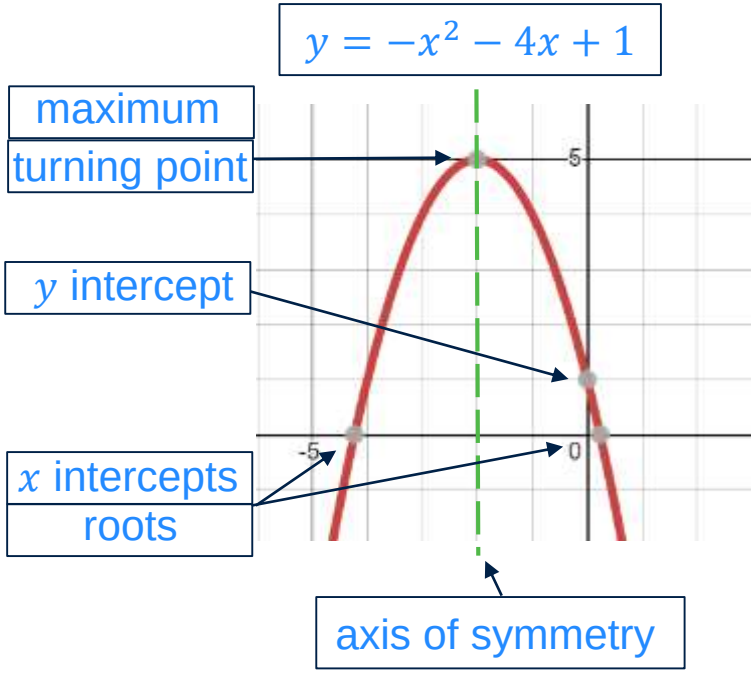
Label it!

Solutions on the next slide….

Label it! Solution

These diagrams show the key features of a quadratic graph

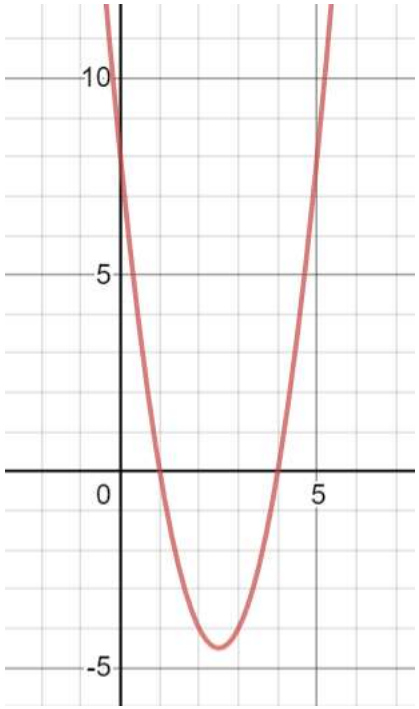


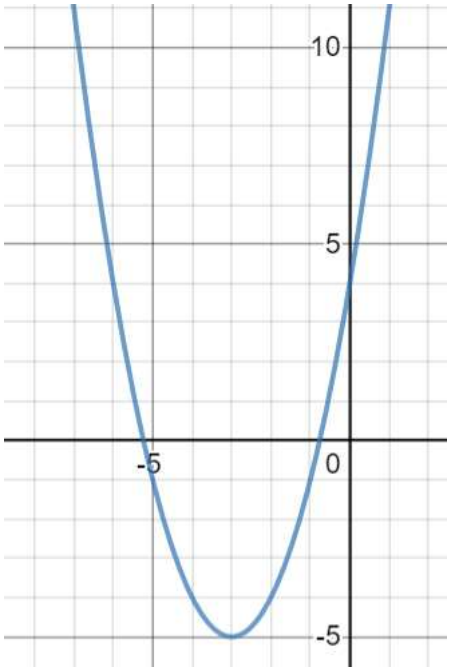


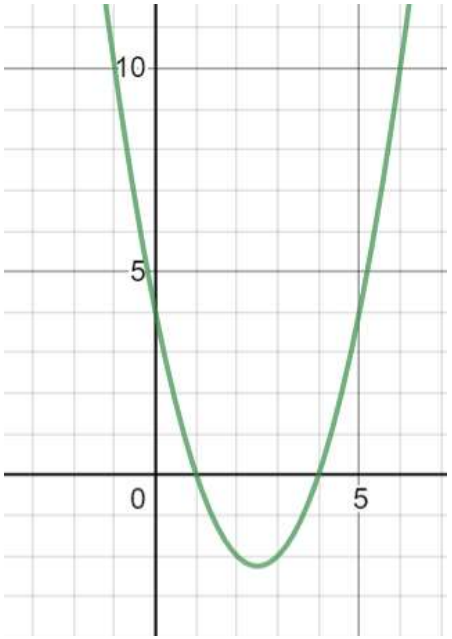
N.B. In mathematics a sketch does not need to be a completely accurate drawing, but it must illustrate all the significant features of the graph/shape.

Identification Parade

- Which of the following graphs is $y = x^2 - 5x + 4$







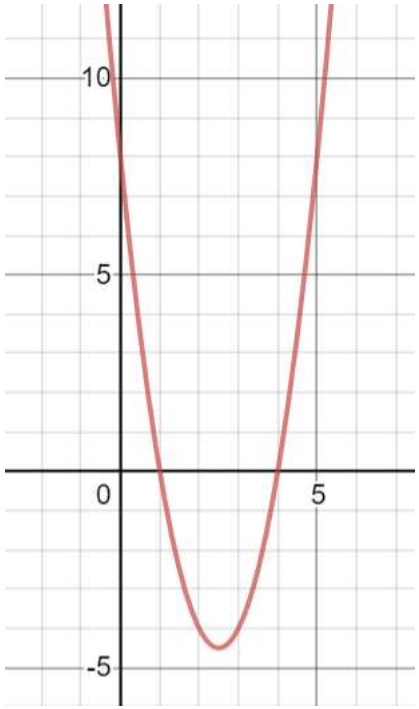
Identification Parade

Solutions on the next slide….

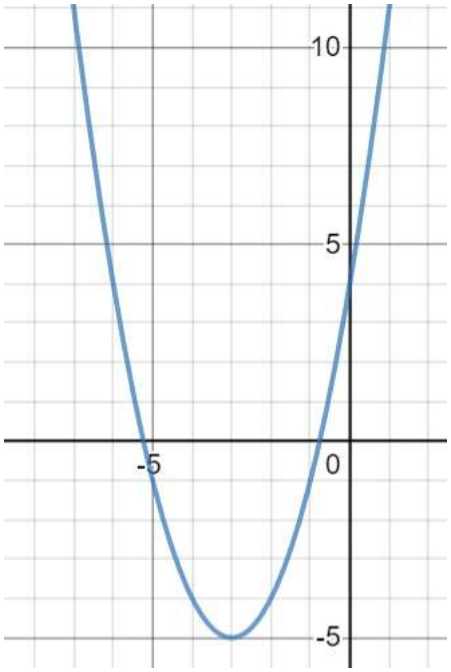
Identification Parade Solution

$$y = x^2 - 5x + 4$$
$$\Rightarrow y = (x - 1)(x - 4)$$

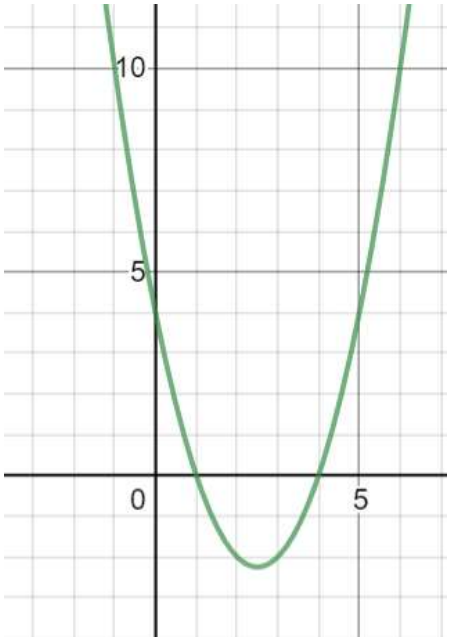
So the correct graph has x intercepts (1,0) and (4,0) and y intercept (0,4)



The red graph has the correct x intercepts but it has a y intercept at (0,8)



The blue graph has the correct y intercept only



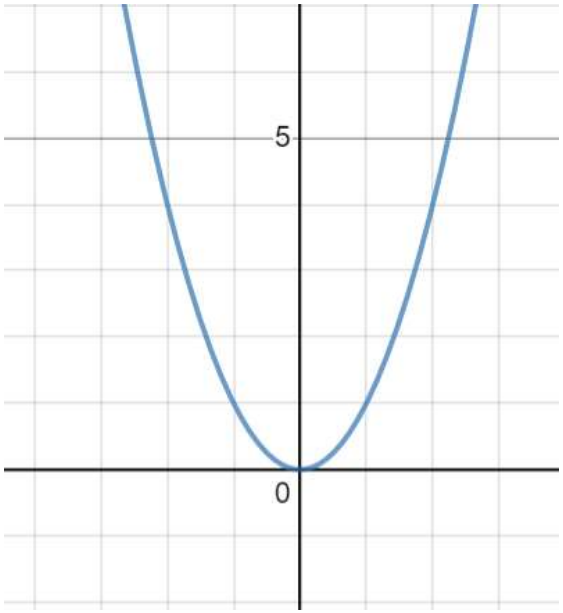
The graph above is $y = x^2 - 5x + 4$

Move it!

Look at the two graphs below.

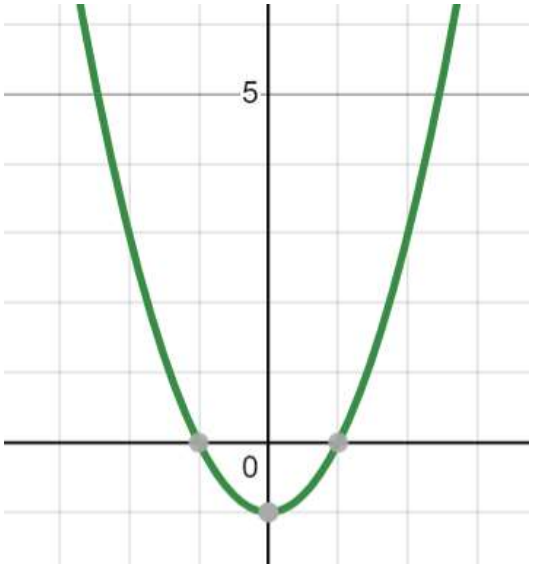
- Can you describe how to move Graph A onto Graph B?

GRAPH A



$y = x^2$

GRAPH B



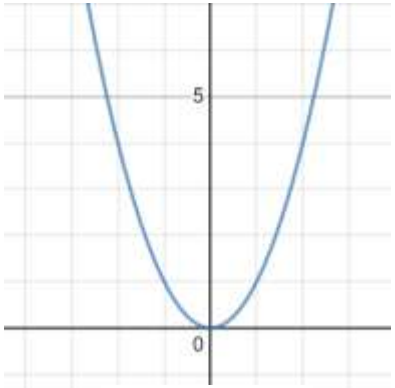
$y = x^2 - 1$

- Can you see how that links to the equation of the graph?

Move it!

- Can you describe how to move Graph A onto Graph B?

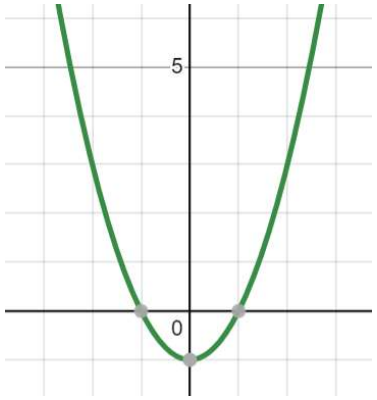
GRAPH A



$y = x^2$

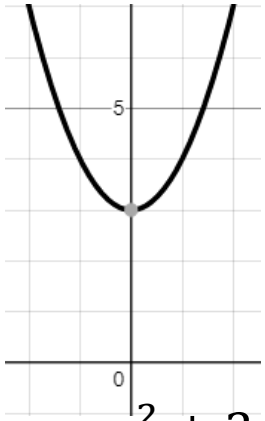
Translate by the vector $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$

GRAPH B

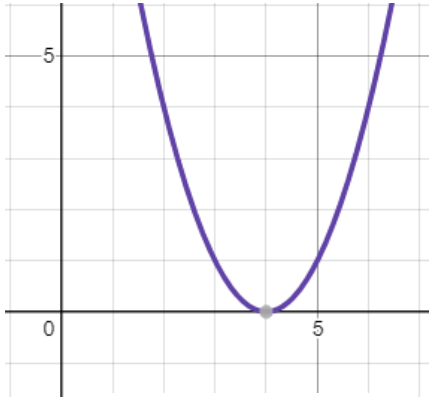


$y = x^2 - 1$

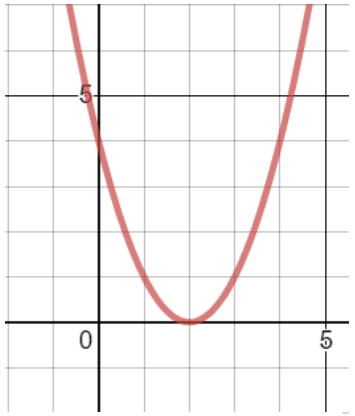
- Which transformations would take GRAPH A onto each of the graphs below?



$y = x^2 + 3$



$y = (x - 4)^2$



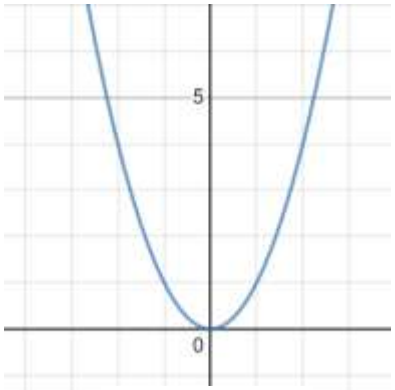
$y = (x - 2)^2$

Move it!

Solutions on the next slide….

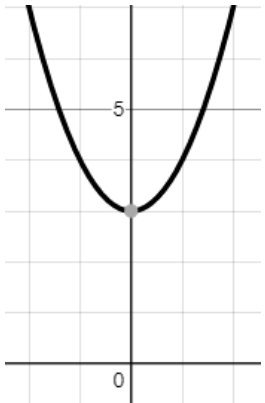
Move it! Solutions

GRAPH A



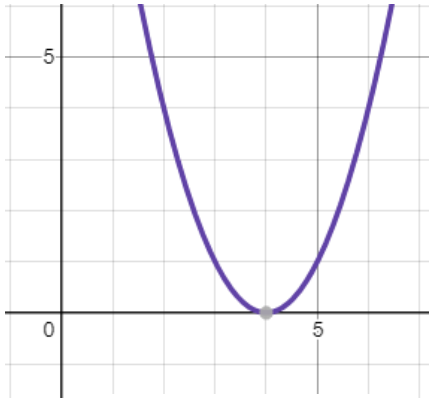
$y = x^2$

Which transformations would take GRAPH A onto each of the graphs below?



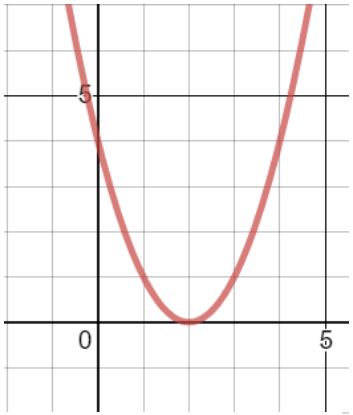
$y = x^2 + 3$

Translate by the vector $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$



$y = (x - 4)^2$

Translate by the vector $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$



$y = (x - 2)^2$

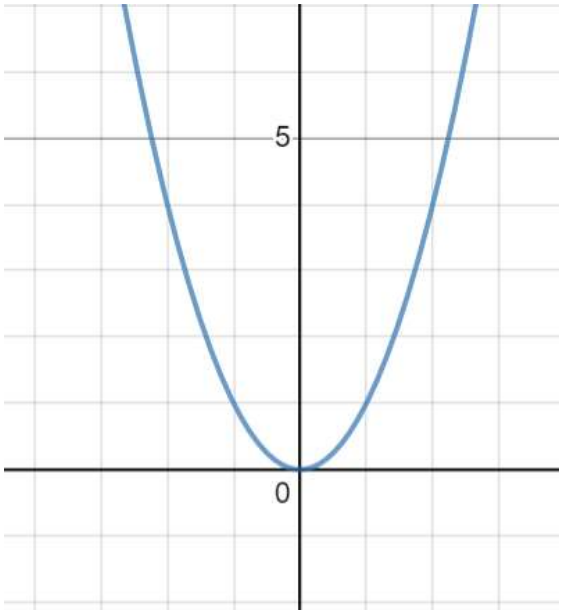
Translate by the vector $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

Move it again!

Look at the new graphs below.

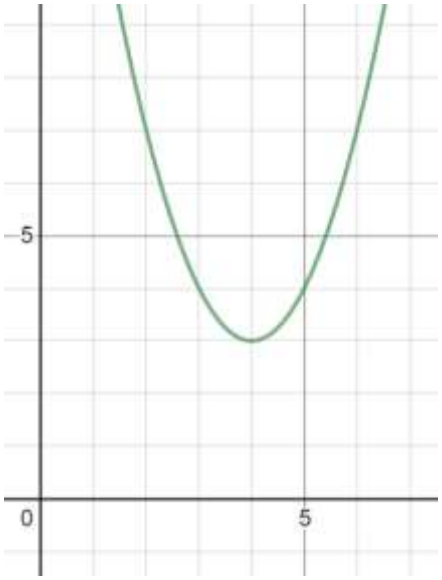
- Can you describe how to move Graph A onto Graph B?

GRAPH A



$y = x^2$

GRAPH B



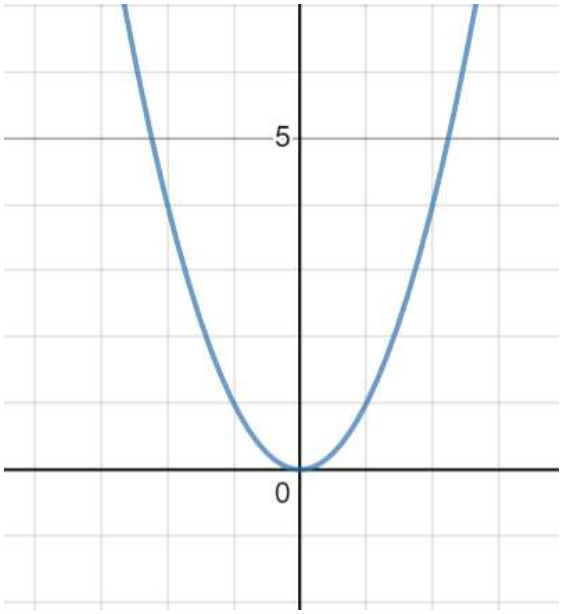
$y = (x - 4)^2 + 3$

- Can you see how that links to the equation of the graph?

Move it again!

- Can you describe how to move Graph A onto Graph B?

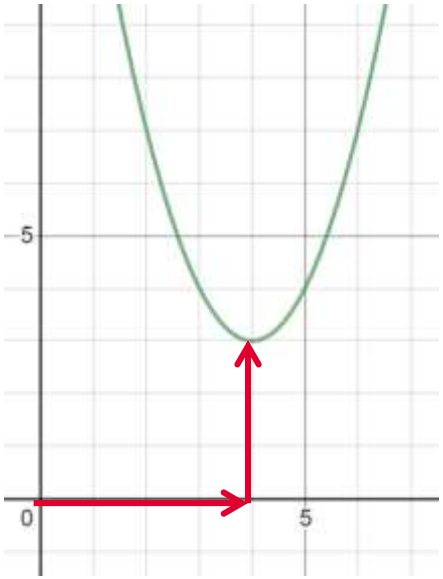
GRAPH A



$y = x^2$

Translate by the vector $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$

GRAPH B



$y = (x - 4)^2 + 3$

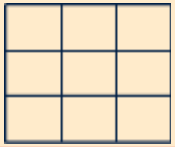
Minimum coordinate at $(4,3)$

Completing the square gives us the coordinate of the minimum point of the graph and it also gives us the translation from $y = x^2$ to another quadratic graph

Completing the square form is very useful for all types of questions involving quadratics and circles at A level

Complete the square to get sorted!

Draw out a 3 by 3 grid like this one that's large enough to write the equations in



You have been given 10 equations. Your task is to place 9 of them onto the grid according to these rules on the cards

The turning point of the equation $y = (x + a)^2 + b$ is at $(-a, b)$

All of the coordinates of the turning points are at integer values of x and y. None of the turning points are on either axis.

All of the equations in the top row have a turning point on the line $y = 4$

One of the equations in the left hand column has its turning point at (-4, 4)

The equation with a turning point at (-1, 4) is not on the top row

The equations in the top left and centre right square both have the same y coordinate for their turning point

All three of the equations with a turning point on the line $x = 3$ are on the bottom row

All of the equations in the centre column have turning points on the line $y = 5 - x$

All of the turning points for the equations in the centre column are in the first quadrant

$y = x^2 + 6x + 10$ is in the square in the left hand column directly above $y = x^2 - 6x + 16$

The equations to sort are:

$y = x^2 - 6x + 16$

$y = x^2 - 6x + 25$

$y = x^2 - 4x + 7$

$y = x^2 + 2x + 5$

$y = x^2 - 6x + 11$

$y = x^2 - 8x + 21$

$y = x^2 - 2x + 5$

$y = x^2 + 8x + 20$

$y = x^2 + 6x + 10$

$y = x^2 - 10x + 29$

Complete the square to get sorted!

Solutions on the next slide….

Complete the square to get sorted! Solution

$y = x^2 + 8x + 20$ $y = (x + 4)^2 + 4$ $(-4,4)$	$y = x^2 - 2x + 5$ $y = (x - 1)^2 + 4$ $(1,4)$	$y = x^2 - 10x + 29$ $y = (x - 5)^2 + 4$ $(5,4)$
$y = x^2 + 6x + 10$ $y = (x + 3)^2 + 1$ $(-3,1)$	$y = x^2 - 4x + 7$ $y = (x - 2)^2 + 3$ $(2,3)$	$y = x^2 + 2x + 5$ $y = (x + 1)^2 + 4$ $(-1,4)$
$y = x^2 - 6x + 16$ $y = (x - 3)^2 + 7$ $(3,7)$	$y = x^2 - 6x + 11$ $y = (x - 3)^2 + 2$ $(3,2)$	$y = x^2 - 6x + 25$ $y = (x - 3)^2 + 16$ $(3,16)$

All turning points on the line $y = 4$ But not $(-1,4)$

All turning points on the line $x = 3$

One equation has turning point $(4,4)$

All turning points on the line $y = 5 - x$ and in first quadrant

This is the equation that doesn't fit in the table

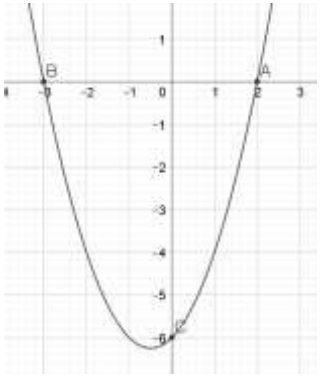
$y = x^2 - 8x + 21$

Quadratic Graphs 2

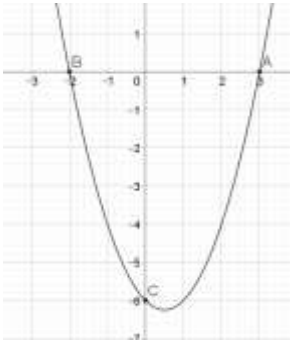
1. What are the x intercepts of

$$y = (2x + 3)(x + 4)$$

2. What are the x and y intercepts of this graph.



3. Write the equation of the graph in the form $ax^2 + bx + c$

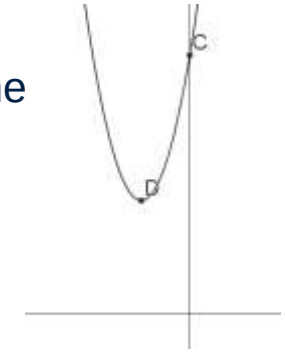


4. What are the x intercepts of the graph of $y = 6x^2 + x - 2$

5. What does the c part of the equation in $y = ax^2 + bx + c$ represent on a graph?

6. Sketch the graph of $y = 3x^2 - 2x - 8$. Label x and y intercepts

7. What are the co-ordinates of the points marked on the diagram of the equation $y = x^2 + 6x + 16$



8. Which of these statements about the graph $y = x^2 - 4x + 8$ are true

Has a minimum point at (2, 4)	**Will not cross the x axis twice**	**Can be factorised**

Quadratic Graphs 2

Solutions on the next slide….

Quadratic Graphs 2 Solutions

1. What is the x intercept of

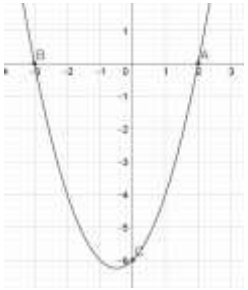
$$y = (2x + 3)(x + 4)$$

$$(2x + 3)(x + 4) = 0$$

$$x = -\frac{3}{2} \quad x = -4$$

x intercepts are $\left(-\frac{3}{2}, 0\right)$ and $(-4,0)$

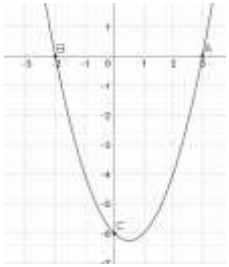
2. What are the x and y intercepts of this graph.



x intercepts are $(-3, 0)$ and $(2,0)$

y intercepts is $(0, -6)$

3. Write the equation of the graph in the form $ax^2 + bx + c$



x intercepts are $(-2, 0)$ and $(3,0)$
which come from $(x + 2)(x - 3) = 0$
expanding gives

$$x^2 - x - 6 = y$$

4. What are the x intercepts of the graph $y = 6x^2 + x - 2$

$$6x^2 + x - 2 = 0$$

$$(3x + 2)(2x - 1) = 0$$

x intercepts are $\left(-\frac{2}{3}, 0\right)$ and $\left(\frac{1}{2}, 0\right)$

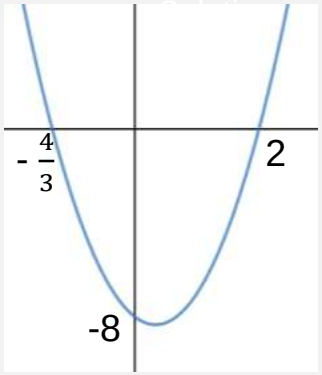
Quadratic Graphs 2 Solutions

5. What does the c part of the equation $y = ax^2 + bx + c$ represent on a graph?

This gives the y-intercept (when $x = 0$) so the y intercept is $(0, c)$

6. Sketch the graph of $y = 3x^2 - 2x - 8$. Label x and y intercepts

Factorise to get x intercept
$(3x + 4)(x - 2) = 0$
$\left(-\frac{4}{3}, 0\right)$ and $(2,0)$
y intercept $(0,-8)$



7. What are the co-ordinates of the points marked on the diagram of the equation $y = x^2 + 6x + 16$

C is y intercept $(0,16)$
D is found by completing the square
$y = (x + 3)^2 + 7$
D is $(-3,7)$

8. Which of these statements about the graph $y = x^2 - 4x + 8$ are true

By completing the square we get $(x - 2)^2 + 4$

Has a minimum point at (2, 4)

Equation does not factorise x^2 is positive so 'U' shaped

Will not cross the x axis twice

Has a minimum point at (2, 4)

Will not cross the x axis twice

Can be factorised

How High?

The height of a ball thrown up from the ground into the air at time, t, is given by:

$$h = 20t - 10t^2$$

- Find when the ball hits the ground
- How long is the ball more than 5m above the ground?
- Find the maximum height reached by the ball

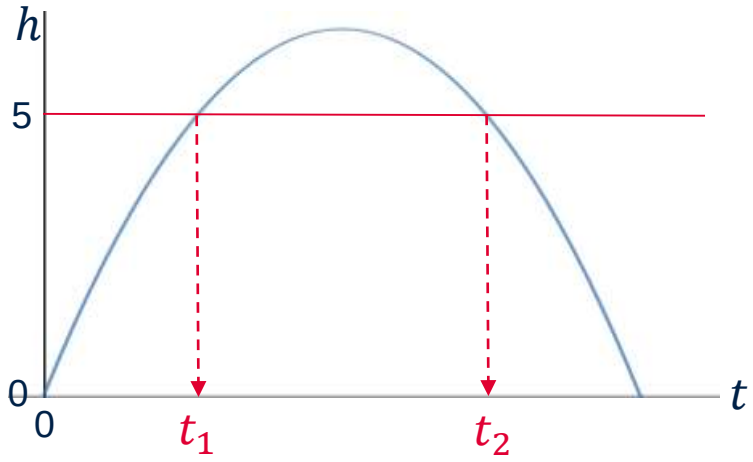
How High?

Solutions on the next slide….

How High? Solution

The height of a ball thrown up from the ground into the air at time, t, is given by:

$$h = 20t - 10t^2$$



- Find when the ball hits the ground

When the ball hits the ground the height $h = 0$

$$20t - 10t^2 = 0$$
$$10t(2 - t) = 0$$
$$t = 0 \text{ or } t = 2$$

(the ball is launched at $t = 0$ and returns to the ground at $t = 2$)

- How long is the ball more than 5m above the ground?

The ball is above 5m when $h > 5$

Rearrange and solve using the quadratic formula

$$20t - 10t^2 > 5$$
$$-10t^2 + 20t - 5 = 0$$
$$t = 1 \pm \frac{\sqrt{2}}{2}$$

$t_1 = 0.293$ and $t_2 = 1.707$ $(to\ 3dp)$

So the time above 5m is $t_2 - t_1 = 1.414$ seconds

- Find the maximum height reached by the ball

Max height of the ball is at the turning point

You could write expression in completed square form

Or, by symmetry, when $t = 1$

$$h = 20 \times 1 - 10 \times 1^2 = 10$$

So maximum height is 10 metres

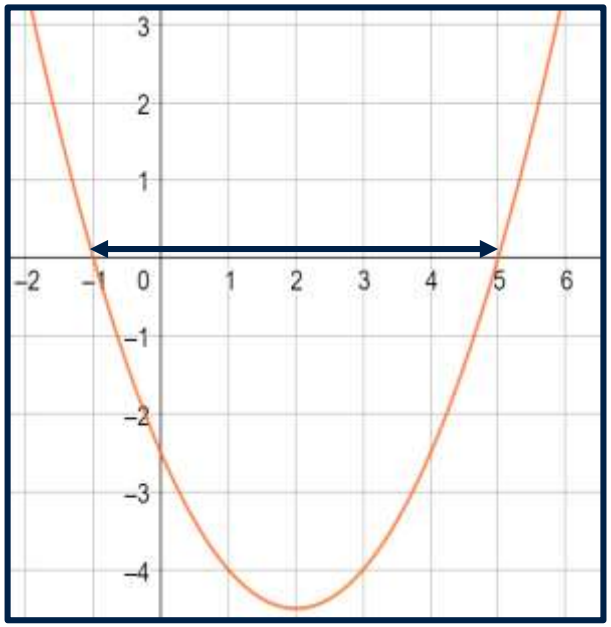
Inequalities reminder

Solve the inequality $x^2 - 4x - 5 < 0$

Rearrange into factorised form…

$$(x - 5)(x + 1) < 0$$

…now you can sketch the graph



When $x > -1$ and $x < 5$
the curve is below the x axis.

This is where $x^2 - 4x + 5 < 0$

This is one region, so can be represented by one inequality

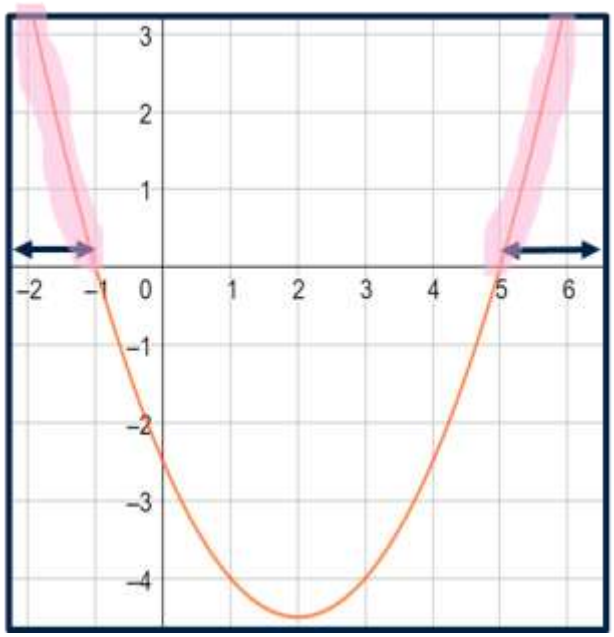
$$-1 < x < 5$$

Inequalities reminder

Solve the inequality $x^2 - 4x - 5 \geq 0$

Rearrange into factorised form…

$$(x - 5)(x + 1) \geq 0$$



…now you can sketch the graph

When $x < -1$ and $x > 5$
the curve is above the x axis.

This is where $x^2 - 4x + 5 \geq 0$

These are two regions, so are represented by two inequalities

$x < -1$ and $x > 5$

Quadratic Inequalities

Use a sketch to help you solve the following inequalities

1. $(x-2)(x+3) < 0$

2. $(4+x)(2-x) < 0$

3. $x^2 + 7x + 12 \geq 0$

4. $36 \geq (x+2)^2$

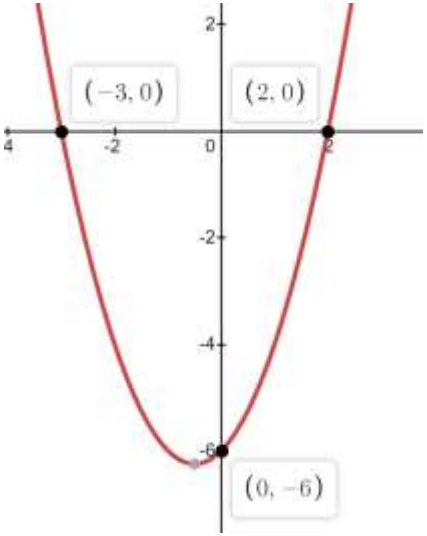
Quadratic Inequalities

Solutions on the next slide….

Quadratic Inequalities Solutions

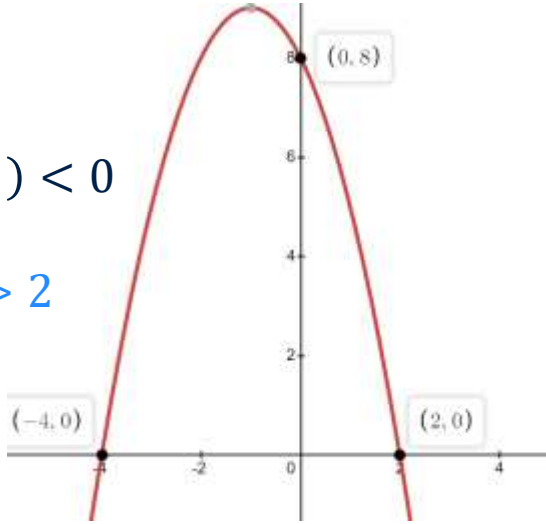
1. $(x-2)(x+3)<0$

$-3<x<2$



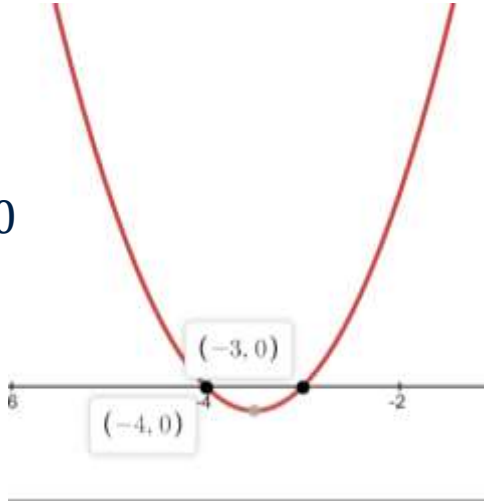
2. $(4+x)(2-x)<0$

$x<-4$ or $x>2$



3. $x^2+7x+12\geq 0$

$x\leq -4$ or $x\geq -3$

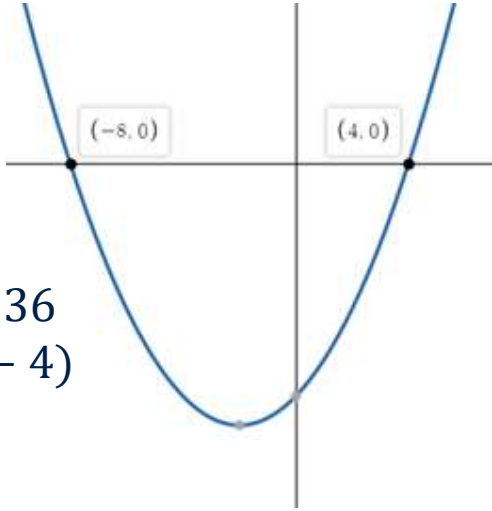


4. $36\geq (x+2)^2$

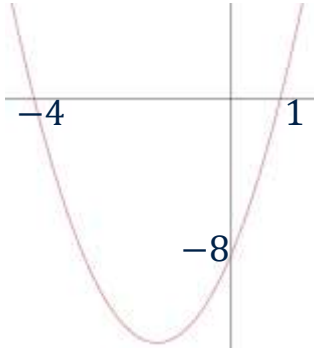
$0\geq (x+2)^2-36$

$0\geq (x+8)(x-4)$

$-8\leq x\leq 4$



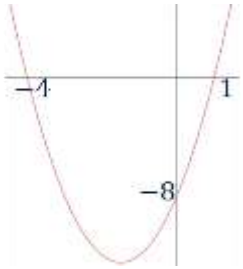
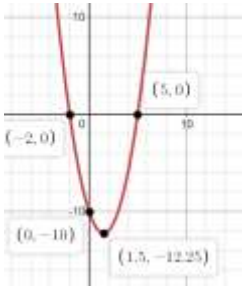
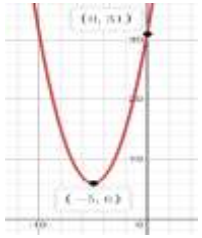
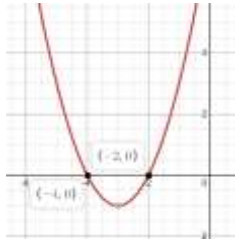
Fill the table

Sketch	Equation	x intercept	y intercept	Minimum point
				
		$(5,0)$ $(-2,0)$	$(0,-10)$	
				(-5, 6)
	$y = x^2 + 6x + 8$		$(0,8)$	

Fill the table

Solutions on the next slide….

Fil the table Solutions

Sketch	Equation	x intercept	y intercept	Minimum point
	$y = (x+4)(x-1)$ $y = x^2 + 3x - 4$	$(-4,0)(1,0)$	$(0,-4)$	$\left(x+\frac{3}{2}\right)^2 - \frac{9}{4} - 4$ $\left(x+\frac{3}{2}\right)^2 - \frac{25}{4}$ Min point $(-\frac{3}{2}, -\frac{25}{4})$
	$y = (x-5)(x+2)$ $y = x^2 - 3x - 10$	$(5,0)\ (-2,0)$	$(0,-10)$	$\left(x-\frac{3}{2}\right)^2 - \frac{9}{4} - 10$ $\left(x-\frac{3}{2}\right)^2 - \frac{49}{4}$ Min point $(\frac{3}{2}, -\frac{49}{4})$
	$y = (x+5)^2 + 6$ $y = x^2 + 10x + 31$	Sits above x axis so no intercepts	$(0,31)$	(-5, 6)
	$y = x^2 + 6x + 8$	$(-4,0)(-2,0)$	$(0,8)$	$(x+3)^2 - 9 + 8$ $(x+3)^2 - 1$ Min point $(-3,-1)$

Catching Stars

Click here to try a Quadratic Marbleslides Challenge

You will be investigating the features of quadratic graphs whilst trying to catch as many stars as possible

You can join the activity without signing in or entering your real name.

Still want more?

Challenge yourself to find the mystery graphs with this game from MEI. Will you take your time or compete against the clock?

Discover more about parabolas and their use, including their use in mirrors and satellite dishes.

Watch this video and learn how getting your paraboloid wrong can have some very unintended consequences! The video includes a little bit of A Level maths content.

Contact the AMSP

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Advanced_Maths