Advanced Mathematics Support Programme®

Did you know?

Trigonometric functions can be used to model many things that repeat over a time period, for example

Tides

Springs

Harmonic Strings

Daylight

Sketching Other Graphs 1

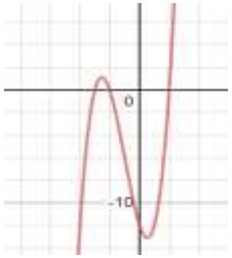
1. What is the mathematical name for the graph of $y = \frac{1}{x}$?

2. What are the maximum and minimum values for the graph $y = cos\theta$?

3. Sketch the graph of $y = 2^x$. Label the y and x intercepts

4. Using a sketch of the graphs
$$y = \frac{1}{x} \text{ and } y = x$$
show how many solutions there will be to the equation $\frac{1}{x} = x$

5. What is the name for this type of graph?



6. What is the y intercept of the graph $y = (x + 2)(x - 3)(x + 5)$?

7. What are the x intercepts of the graph $y = (x + 2)(x - 3)(x + 5)$?

8. Sketch the graph of $y = (x - 3)(x + 2)(x + 5)$

Sketching Other Graphs 1

Solutions on the next slide….

Sketching Graphs 1 Solutions

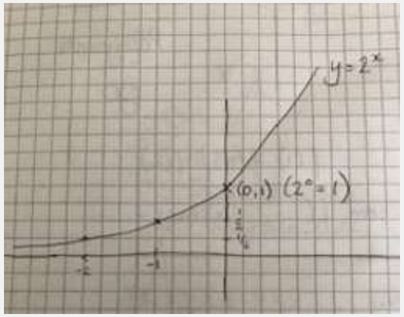
1. What is the mathematical name for the graph of $y = \frac{1}{x}$?

→ A reciprocal graph

2. What are the maximum and minimum values for the graph $y = cos\theta$?

→ Max value = 1
Min value = -1

3. Sketch the graph of $y = 2^x$.
Label the y and x-intercepts

→ 

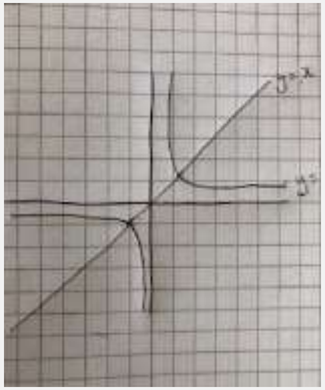
As x gets very large y gets very large

As x gets very small, y tends to zero but stays positive

4. Using a sketch of the graphs

$$y = \frac{1}{x} \text{ and } y = x$$

show how many solutions there will be to the equation $\frac{1}{x} = x$

→ 

There will be two solutions

Sketching Graphs 1 Solutions

5. What is the name for this type of graph? → A cubic

6. What is the y intercept of the graph $y = (x+2)(x-3)(x+5)$? →

y intercept when $x = 0$

$y = (0+2)(0-3)(0+5)$

$y = 2 \times (-3) \times 5$

$y = -30$

y intercept at $(0, -30)$

7. What are the x intercepts of the graph $y = (x+2)(x-3)(x+5)$? →

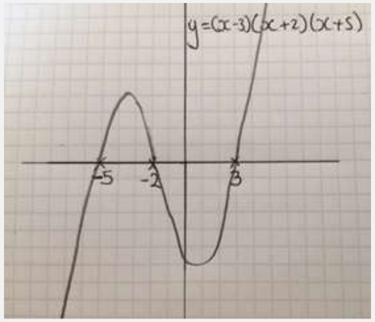
x intercepts when $y = 0$

$0 = (x+2)(x-3)(x+5)$

$x = -2$ and $x = 3$ and $x = -5$

x intercepts at $(-2,0)$ $(3,0)$ $(-5,0)$

8. Sketch the graph of $y = (x-3)(x+2)(x+5)$ →



Unsure about any of these? Search Identifying Graphs. Next try Sketching graphs 2….

Sketching Graphs 2

1. What is the mathematical name for graphs of the form of $x^2 + y^2 = 9$?

2. Sketch the graph of $y = sin\theta$ between $0°$ and $360°$, labelling x and y intercepts

3. On your sketch for Q2 draw in the line
$$y = 0.5$$
How many solutions are there to
$$sin\theta = 0.5?$$
Can you say what they are?

4. Sketch the graph $y = x^3$, labelling any intersections

5. Sketch the graph of the equation in Q1, label any intersections with the x and y axis

6. What is the y intercept of the graph $y = (x + 1)(x + 1)(x - 1)$?

7. What are the x intercepts of the graph $y = (x + 1)(x + 1)(x - 1)$?

8. Sketch the graphs of
$$x^2 + y^2 = 4$$
$$y = x + 1$$
Use the sketch to determine how many solutions there are when those equations are solved simultaneously

Sketching Graphs 2

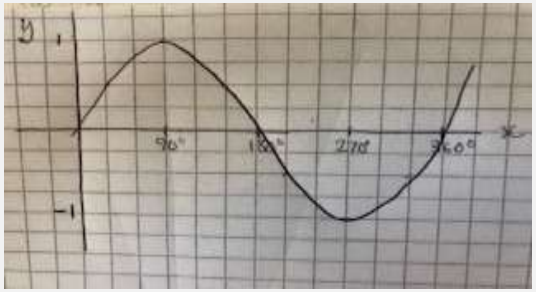
Solutions on the next slide….

Sketching Graphs 2 Solutions

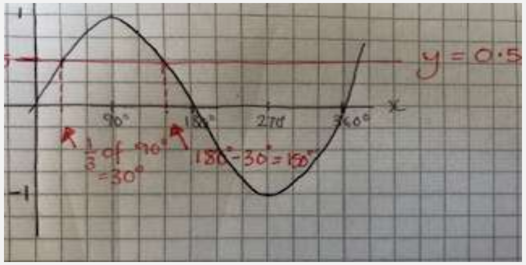
1. What is the mathematical name for graphs of the form of $x^2 + y^2 = 9$?

Circles are of the form $x^2 + y^2 = r^2$

2. Sketch the graph of $y = sin\theta$ between $0°$ and $360°$, labelling x and y intercepts

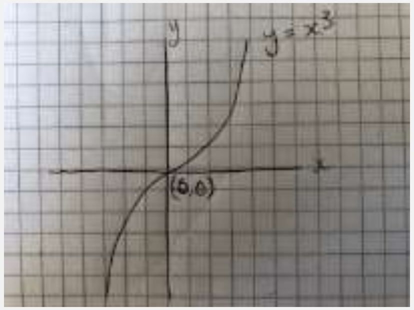


3. On your sketch for Q2 draw in the line
$$y = 0.5$$
How many solutions are there to
$$sin\theta = 0.5?$$
Can you say what they are?



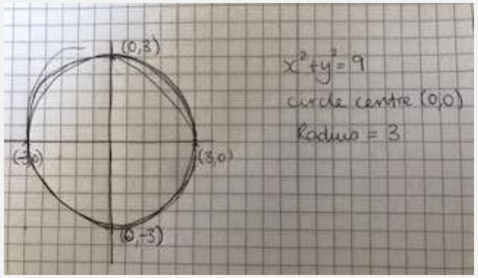
Two solutions
30° and 150°
The points of intersection

4. Sketch the graph $y = x^3$, labelling any intersections



Sketching Graphs 2 Solutions

5. Sketch the graph of the equation in Q1. Label any intersections with the x and y axis



6. What is the y-intercept of the graph $y = (x + 1)(x + 1)(x - 1)$?

y intercept when $x = 0$

$$y = (0 + 1)(0 + 1)(0 - 1)$$
$$y = 1 \times 1 \times -1$$
$$y = -1$$

y intercept is $(0, -1)$

7. What are the x intercepts of the graph $y = (x + 1)(x + 1)(x - 1)$?

x intercept when $y = 0$

$0 = (x + 1)(x + 1)(x - 1)$

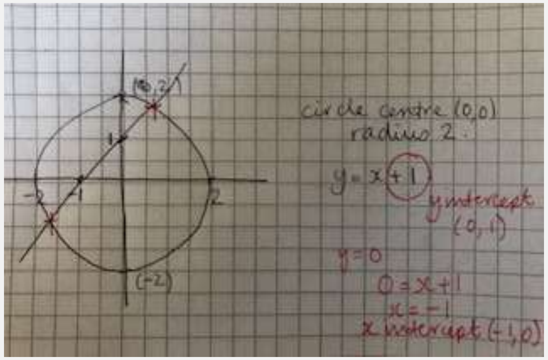
$$x = -1 \quad x = 1$$

x intercept is $(-1,0)$ repeated and $(1, 0)$

8. Sketch the graphs of

$$x^2 + y^2 = 4$$
$$y = x + 1$$

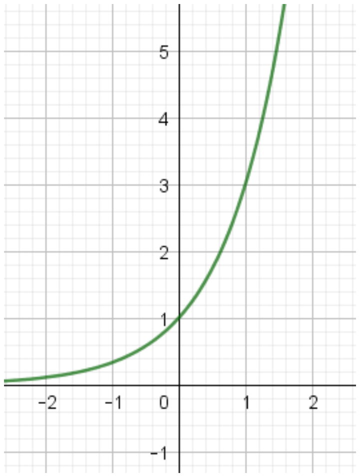
Use the sketch to determine how many solutions there are when those equations are solved simultaneously

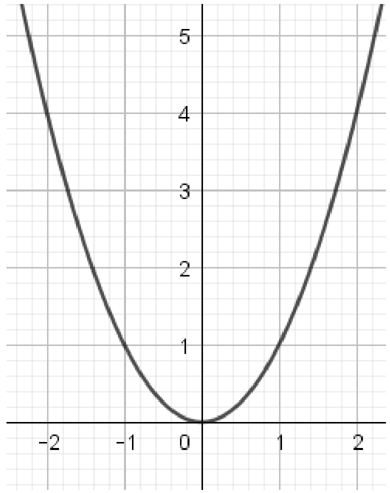


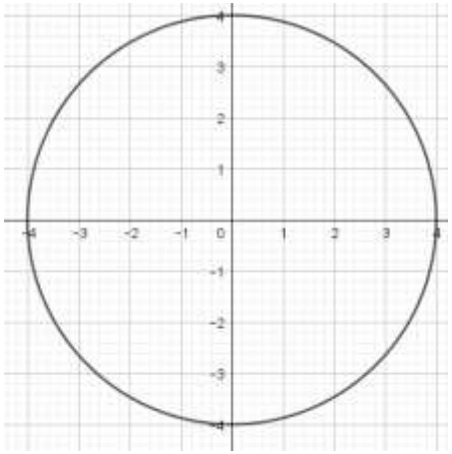
Two solutions

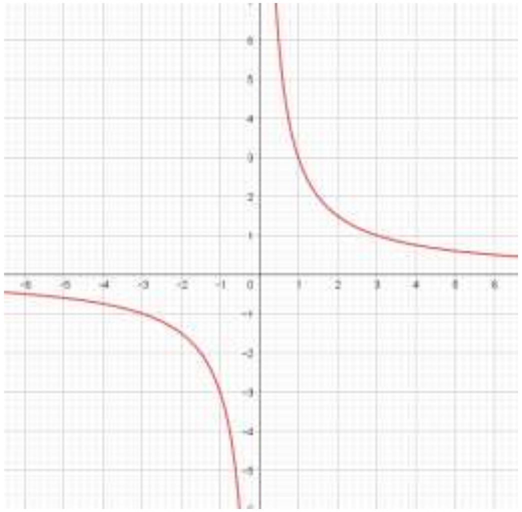
Which is which?

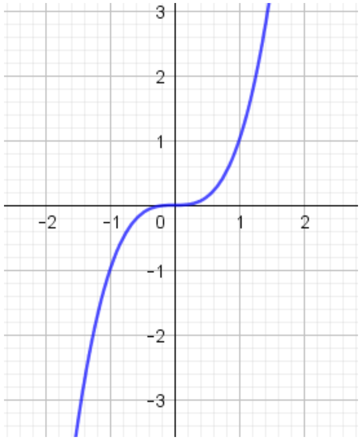
Match the graphs to the equations There are more equations than you need!

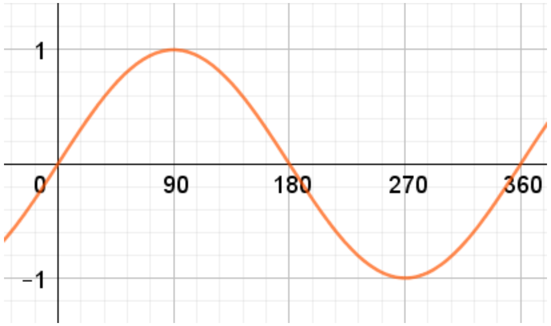












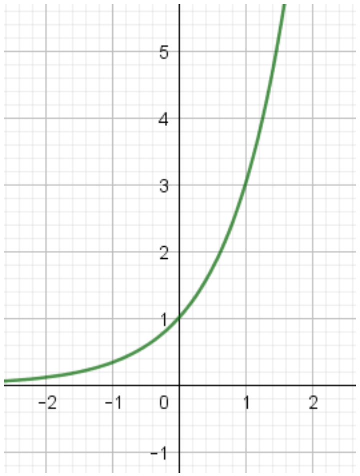
- $y = 3^x$
- $y = \frac{3}{x}$
- $y = sin\theta$
- $y = 2x^2$
- $y = tan\theta$
- $y^2 + x^2 = 4$
- $y = x^2$
- $y = x^3$
- $y = \frac{1}{x}$
- $y = x^2 + 3$

Which is which?

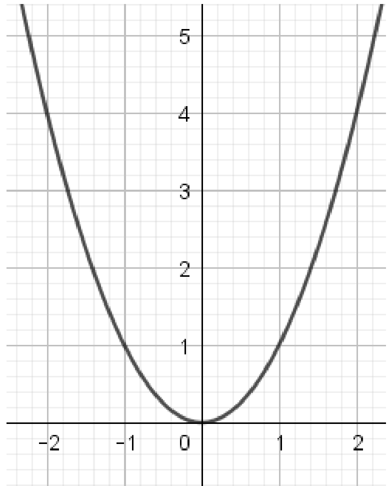
Solutions on the next slide….

Which is which?

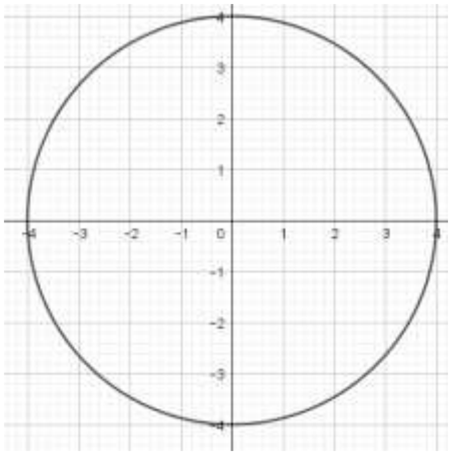
Match the graphs to the equations



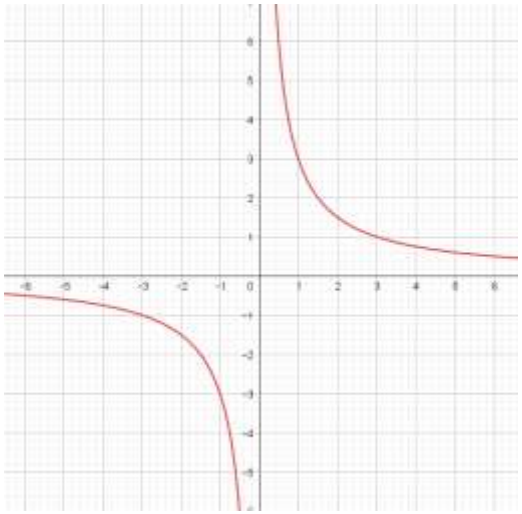
$y = 3^x$



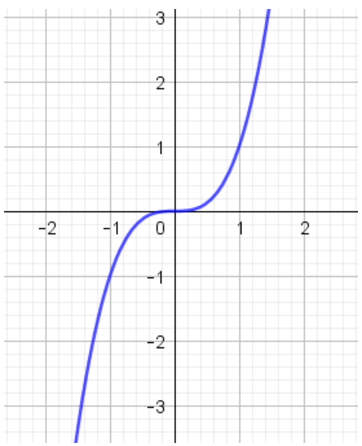
$y = x^2$



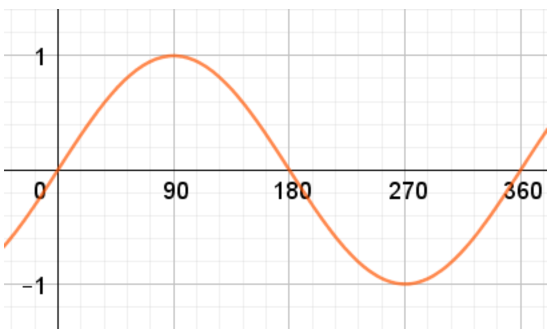
$y^2 + x^2 = 16$



$y = \frac{3}{x}$



$y = x^3$



$y = \sin(x)$

Sketching more than graphs

Find the shortest distance between the following curves:

$$x^2 + y^2 = 9$$
$$y = x^2 + 7$$

A car is initially travelling at 300m/min, it speeds up over a 20 second interval with a constant acceleration to achieve a speed of 400m/min.
It travels at this speed for 3 minutes before slowing to a stop via constant de-acceleration over a period of 30 seconds.

a) What is the car's average speed for the first 20 seconds of travel?
b) What is the car's deceleration?

A square is placed inside a circle (C_1) so that the corners perfectly touch the circle's circumference.
Another circle (C_2) is now placed inside this square so that its circumference perfectly touches the square's sides.

What is the ratio of the lengths of the radius of C_1 and the radius of C_2 ?

Hint: Assume C_2 has a radius of 1 unit

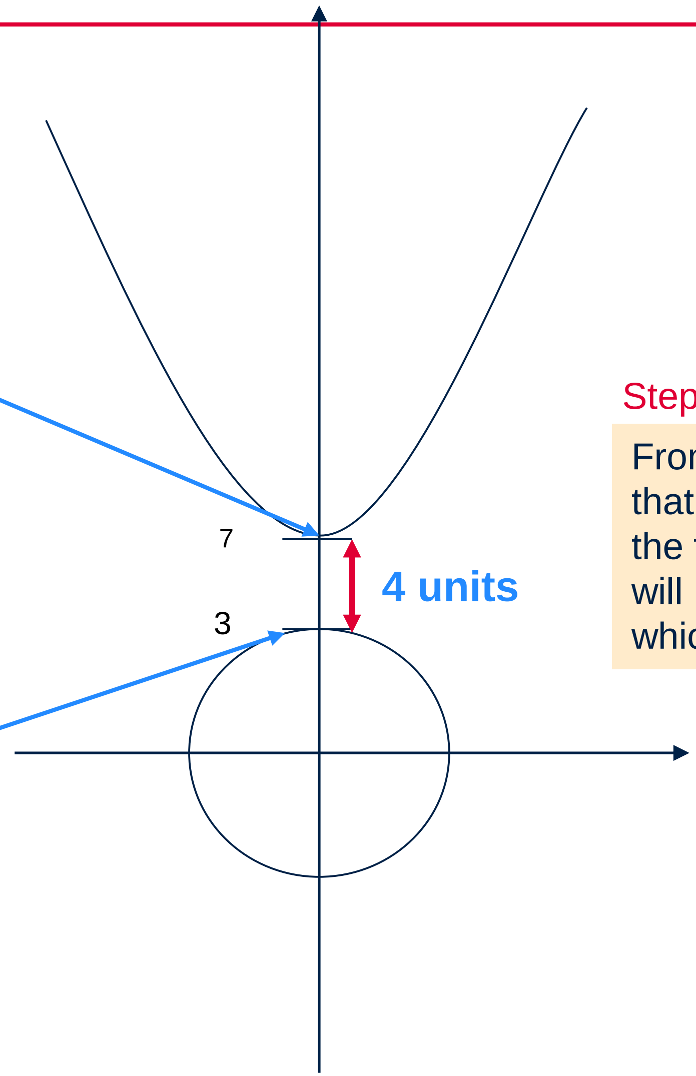
Sketching more than graphs

Solutions on the next slides….

Sketching more than graphs Solution

Find the shortest distance between the following curves: $x^2 + y^2 = 9$

$y = x^2 + 7$



Step 1

The equation $y = x^2 + 7$ is a quadratic and is the equation $y = x^2$ shifted up the y axis by 7 units so lowest point will be at $(0,7)$

Step 2

The equation $x^2 + y^2 = 9$ is a circle with radius 3 and centre (0,0) so sketch circle that goes through the points $(3,0), (0,3), (-3,0)$ and $(0,-3)$.

Step 3

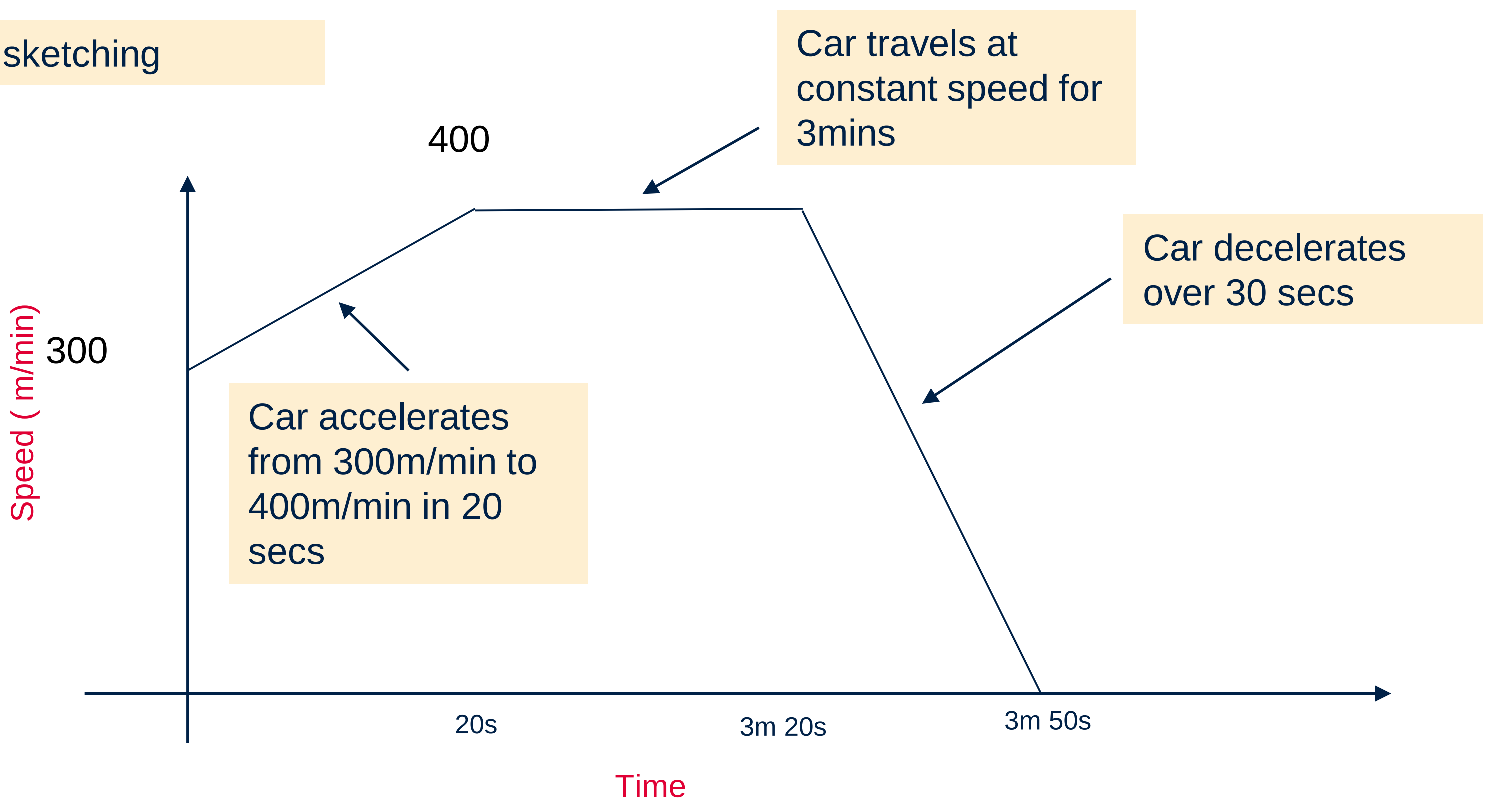
From the 2 sketches we can see that the graphs are separated at the two marked points and this will be the shortest distance, which is 4 units

Sketching more than graphs Solution

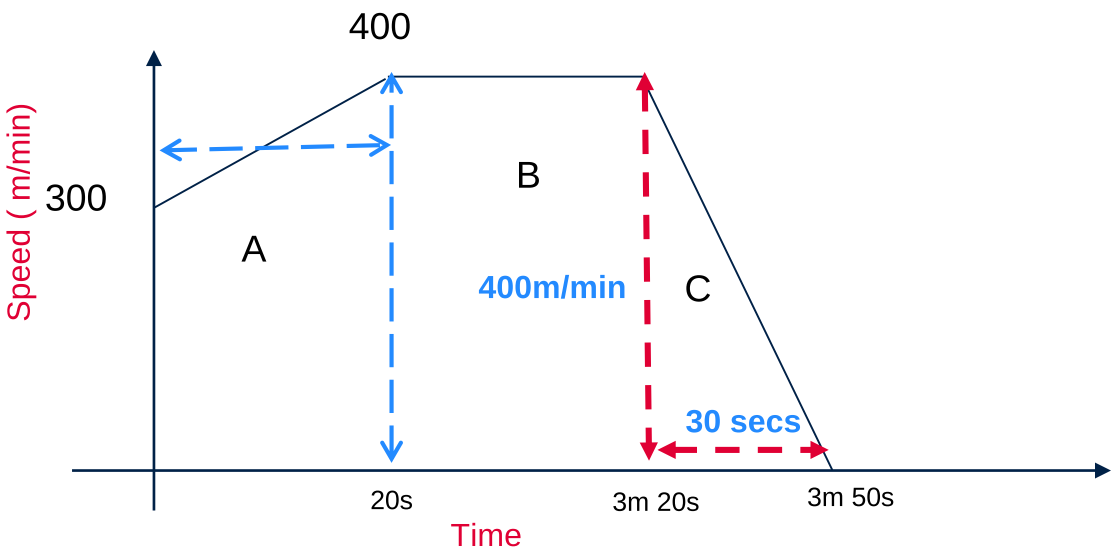
A car is initially travelling at 300m/min, it speeds up over a 20 second interval with a constant acceleration to achieve a speed of 400 m/min.
It travels at this speed for 3 minutes before slowing to a stop via constant de-acceleration over a period of 30 seconds.

Diagram not to scale

Start by sketching



Sketching more than graphs Solution



Note: we need to be careful with units!

a) What is the cars' average speed for the first 20 seconds of travel?

Drawing the blue horizontal line on the graph is a visual way to show that the average speed is half way between 300m/min and 400m/min which is 350m/min

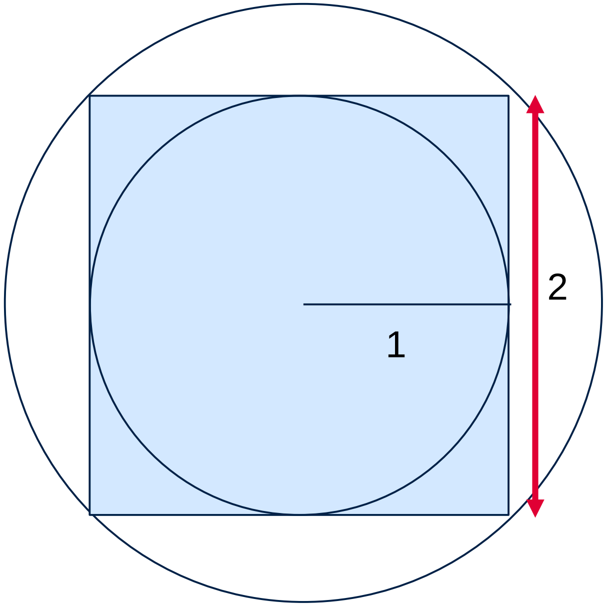
b) What is the cars' deceleration?

By drawing in the red lines we can see that it takes 30 secs for the car to stop - after travelling at 400m/min. (30 seconds = 0.5 minutes), so …
Deceleration = 400 ÷ 0.5 = 800m/min

Sketching more than graphs Solution

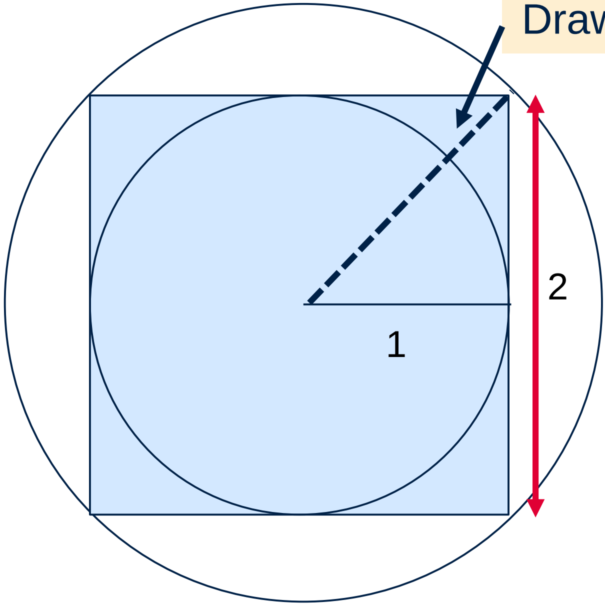
A square is placed inside a circle (C_1) so that the corners perfectly touch the circle's circumference.

Another circle (C_2) is now placed inside this square so that its circumference perfectly touches the square's sides.
What is the ratio of the lengths of the radius of C_1 and the radius of C_2?



Assume inner circle has radius of 1 unit. Therefore square has side length of 2.

Draw in radius of C_1



Using Pythagoras' theorem we can calculate the length of radius of circle 1 to be $\sqrt{2}$.

Therefore the ratio of radii of C_1 to C_2 is:

$$\sqrt{2} : 1$$

Challenge ahead!

The activities on the next few slides may contain some content from A level maths; for that reason they are optional, but still fun and worth trying!

A Triggy Problem

Solve $(sin\,x + 1)(2cosx\ - 1) = 0$ for $0 < x < 360°$

A Triggy Problem

Solutions on the next slide….

Triggy Problems Solutions

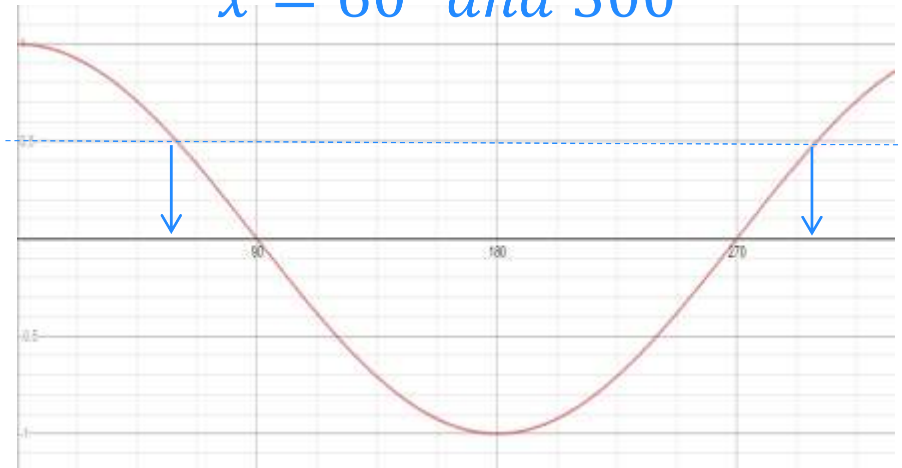
Solve $(\sin x + 1)(2\cos x - 1) = 0$ for $0 < x < 360°$

Fortunately, this is an already factorised quadratic. So…..

$$\sin x + 1 = 0$$
$$\sin x = -1$$
$$x = \sin^{-1}(-1)$$
$$x = 270°$$

or

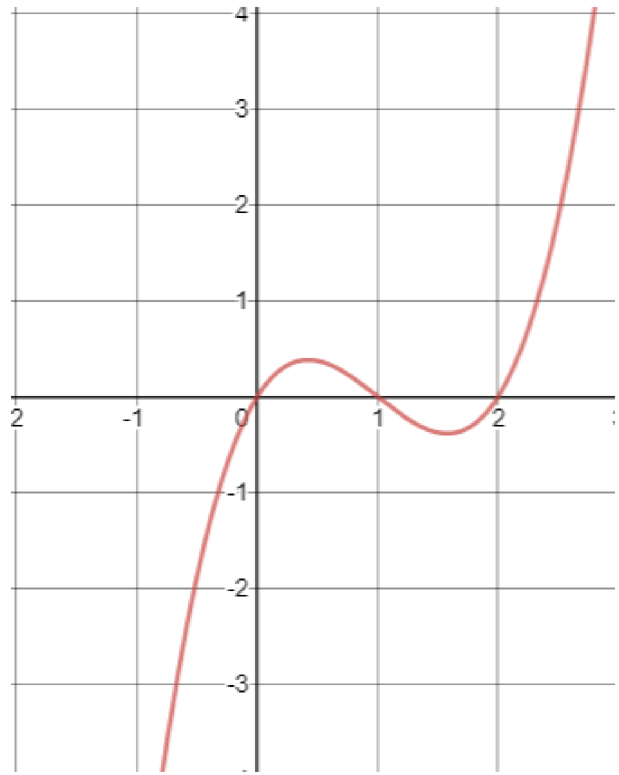
$$2\cos x - 1 = 0$$
$$2\cos x = 1$$
$$\cos x = \frac{1}{2}$$
$$x = \cos^{-1}\left(\frac{1}{2}\right)$$
$$x = 60° \text{ and } 300°$$



A cubic match up

Which one of the equations below describes the graph?

- $y = (x+1)(x-1)(x-2)$
- $y = -x(x-1)(x+1)$
- $y = \ x(x-1)\ (x+1)$

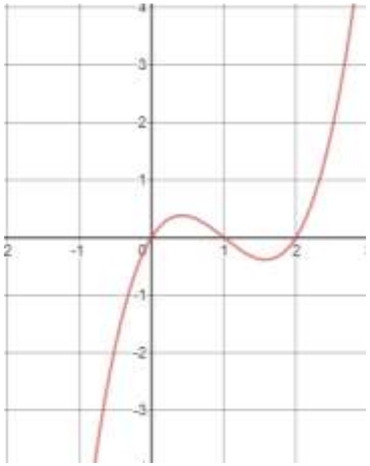


A cubic match up

Solutions on the next slides….

A cubic match up Solution

Which one of the equations below describes the graph?



A negative cubic starts in the top left quadrant and finishes in the bottom right quadrant so it can't be the second equation

Consider when $y = 0$ to find the x intercepts

- $y = (x + 1)(x - 1)(x - 2)$
  $x = -1,\ x = 1,\ x = 2$
- $y = -x(x - 1)(x + 1)$
  $x = 0,\ x = 1,\ x = 1$
- $y = x(x - 1)(x + 1)$
  $x = 0,\ x = 1,\ x = 1$

Both of these equations have the correct intercepts – but which is the correct graph?

The correct equation is $y = x(x - 1)(x + 1)$

Extend what you have learnt about quadratics to help you match up the cubic graphs in this Desmos activity

You can join the activity without signing in or entering your real name.

Learn more about factorising cubics in this activity – with solutions

Catching Stars

Click here to try our Exponential Marbleslides Challenge

You will be investigating the features of exponential graphs whilst trying to catch as many stars as possible

You can join the activity without signing in or entering your real name.

Still want more?

Read about Euclid's Axioms and discover how they might be used in this interactivity. Sketches and diagrams help with more than just questions about graphs!

Play 'Euclidea' to explore more about Euclidean Geometry and constructions.

Watch this video to see how you can 'graph' art! To see all of the 2020 Desmos Art competition finalists (and get inspiration to enter for yourself next year) click here.

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Advanced_Maths